Royalty Stacking and Validity Challenges: The Inverse Cournot Effect *

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Abstract

This paper shows that the well-known royalty-stacking problem is not robust to considering licensors with patents of heterogeneous strength due to the Inverse Cournot effect. The incentives for a downstream producer to challenge a weak patent in court increase when the total royalty rate is lower. The Inverse Cournot effect generates a moderation force in the royalty rate of strong patent holders forcing weak licensors to reduce their royalties to avoid litigation and causing an increase in output. This effect is mitigated when all firms have weak patents, making royalty stacking a more relevant concern in that case.

JEL codes: L15, L24, O31, O34.
keywords: Intellectual Property, Standard Setting Organizations, Patent Licensing, R&D Investment, Patent Pools.

1 Introduction

The fundamental nature of the patent system is under debate among claims on whether it fosters or hurts innovation. The main concerns focus on the impact of patent enforcement in the Information and Communications Technologies (ICT) industry. ICT products, such as laptops, tablets, or smartphones use a variety of technologies covered by complementary patents. The sum of all royalties that must be paid for multiple patented technologies in a single product is said to form a harmful "royalty stack" (Lemley and Shapiro, 2007). This distortion would harm the incentives for firms to invest and innovate in product markets due to the excessively high end-product prices it would entail.

The arguments supporting royalty stacking and the need for a profound reform of the patent system rely on theoretical models which reformulate the well-known problem of Cournot-complements in a licensing framework. In industries where each single product is covered by patents of multiple owners, a patent holder may not fully take into account that an increase in the royalty rate is likely to result in a cumulative rate that may be too high according to other licensors, the licensees, and their customers. Since this negative externality (or Cournot effect) is ignored by all patent holders, the royalty stack may prove inefficiently high.

In this paper we develop a model of the licensing of complementary innovations under the threat of litigation that explains the circumstances under which royalty stacking is likely to be a problem in practice. This model departs from the extant literature in only one natural dimension; we assume that manufacturers of products covered by multiple patented technologies may challenge in court those patents and, crucially, that the likelihood that a judge rules in favor of the patent holder is increasing in the strength of its patent portfolio. This assumption is reasonable. Downstream manufacturers commonly challenge the validity of the patents that cover their products when they litigate in court

the licensing terms offered by patent holders. Those with large and high quality patent portfolios will not be constrained by the threat of litigation when setting their royalty rate. On the contrary, owners of weak portfolios will have to moderate their royalty claims in order to avoid litigation concerning patent validity.

Interestingly, our analysis uncovers a positive relationship between the royalty rate that different patent holders optimally set when facing the threat of litigation from a downstream user of their technology. That is, the ability of a patent holder to charge a high royalty rate without triggering litigation increases in the aggregate royalty rate charged by all other patent holders. The intuition is as follows. A downstream producer will decide to go to court and try to invalidate the patents of an upstream innovator if the increase in profits associated to the corresponding reduction in the aggregate royalty rate compensates for the legal costs involved. Due to the convexity of the profit function with respect to its own costs, this difference is decreasing in the royalty rate set by the remaining patent holders. That is, when the total royalty rate set by all other patent holders is already high and, consequently, profits are low in any case, the marginal gains from invalidating the portfolio of an innovator are also low. This means that the innovator can also charge a higher royalty rate and avoid being challenged in court. This positive relationship is a novel and very general insight that we denote as the *Inverse Cournot effect* and it represents a positive externality among owners of complementary inputs (in this case patents). We show that this effect arises regardless of whether royalty rates are per-unit or ad-valorem or if royalties are offered as part of a two-part tariff.

As a result of the Inverse Cournot effect, royalty rate reductions become more appealing compared to the case where the threat of litigation is ignored. A strong patent holder, by lowering the royalty rate, forces rivals with weaker portfolios to reduce theirs, boosting total production. When the threat of litigation faced by these rivals is significant, the increase in the production is large and it compensates the strong patent holder for the reduced margin from the lower royalty rate. As a result, the Inverse Cournot

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effect becomes a moderating force, partially offsetting the royalty-stacking problem that arises from the Cournot effect. This is in contrast with the case of a single patent holder owning both perfectly complementarity portfolios, who would always be able to charge the monopoly rate as long as one of the patents is sufficiently strong.

This channel becomes less effective, however, among patent holders with weak patent portfolios. To illustrate that result, we consider the case in which a licensee decides to sue patent holders in an endogenous sequence. Because as the portfolio of a patentee is invalidated the aggregate royalty rate goes down, the incentives for the downstream producer to subsequently sue other patent holders become stronger. As a result of this *litigation cascade*, when a weak patentee considers whether to lower the royalty rate it ought to anticipate that, although it might benefit from a smaller royalty stack through an increase in sales, there is also a greater probability of itself being brought to court. The risk of a litigation cascade mitigates the Inverse Cournot effect and, therefore, allows patent holders to sustain a higher royalty rate, sometimes as high as what a single patent holder would choose. Correspondingly, the royalty-stacking problem might be milder when strong and weak patent holders coexist compared to the case where all patent holders are weak

The model is extended in several dimensions. We discuss some features specific to Standard Setting Organization(SSOs), where Standard Essential Patents (SEPs) are licensed on Fair, Reasonable, and Non-Discriminatory (FRAND) terms. We also explore how the analysis can be extended to account for the effect of downstream competition, royalty renegotiation, sequential royalty-rate setting or equilibrium litigation. In all cases, a modified version of the Inverse Cournot effect emerges.

We present the model in section 2 and section 3 characterizes the single innovator benchmark. Section 4 analyzes the case with two innovators and discusses the circumstances under which the Inverse Cournot and the Litigation Cascade effect arise depending on the strength of the portfolio of each firm. Section 5 extends and discusses the

robustness of the results to changing some of the assumptions. Section 6 concludes.

1.1   Literature Review

The existing theoretical literature has observed that the licensing of complementary and essential patents by many developers could give rise to a royalty-stacking problem (Lemley and Shapiro, 2007) and works like Lerner and Tirole (2004) have pointed out that patent pools constitute a natural way to mitigate its effects. It is also known that this distortion is smaller when royalties are paid ad-valorem instead of per-unit (Llobet and Padilla, 2016). There is more debate on its practical relevance in markets like mobile telecommunications, with some works arguing its large effects (Lemley, 2002), while others (Geradin et al., 2007) emphasize that the conditions necessary for its emergence do not typically arise in practice.

Our paper is also related to a long literature on the litigation of probabilistic patents, including papers like Llobet (2003) and Farrell and Shapiro (2008). More recent works have aimed to capture the interaction of these conflicts in contexts like SSOs, analyzing the litigation involving producers and Non-Practicing Entities (NPEs). This is the case, for example, of Choi and Gerlach (2018) that studies the information externalities that arise when a NPE sequentially sues several producers.

The paper closest to ours is Choi and Gerlach (2015). They develop a model in which patent holders with *weak* patents facing the threat of litigation moderate their royalty claims so that the aggregate royalty payment falls below the one that would emerge from a patent pool. In their setup the positive relationship between the royalty rate of both firms arises from a mechanism that differs from the Inverse Cournot effect identified in our paper. If a downstream producer invalidates the patent portfolio of one of the firms, the rival can raise its own royalty rate. This means that the best response of a patent holder to the reduction in the royalty charged for the complementary portfolio of another patentee may be to reduce one's own royalty in order to reduce the likelihood of

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a validity challenge against its own patent. This effect does not arise in our model, since the Inverse Cournot effect occurs even when one of the patentees has *ironclad* patents so that its decision to lower its royalty rate is not driven by the need to avoid going to court.

Bourreau et al. (2015) consider a setup similar to ours to study licensing and litigation in Standard Setting Organizations. The main difference with our paper is that in their setup litigation occurs after production has taken place. As a result, the total quantity produced does not depend on the outcome of this litigation but only on the aggregate royalty rate negotiated ex-ante. This assumption severs the link between the licensing decisions of patent holders and the litigation decisions of licensees, thus eliminating the Inverse Cournot effect that plays a crucial role in our setup.

Finally, this paper is related to the literature on patent holdout (Epstein and Noroozi, 2018). Small innovators might not find it worthwhile to enforce their patent portfolio due to the legal costs involved. In anticipation of that, it is optimal for users of the technology to infringe on those patents unless the royalty rate offered is sufficiently low. As Lichtman (2006) points out, this implies that when complementary patents that cover the same technology are owned by a few firms, the commitment to litigate will be stronger and the royalty stacking problem will be more relevant than when ownership is disperse. Although patent holdout is present in our paper, our model emphasizes how the interaction with other patent holders might affect the incentives to litigate above and beyond this effect. We show that the Inverse Cournot Effect induces even strong patent holders that are unaffected by patent holdout to choose a lower royalty rate. Furthermore, we show that once we account for this effect, weaker portfolios might actually lead to a higher royalty stack.

2 The Model

Consider a market in which a downstream monopolist, firm B, faces a twice-continuously differentiable demand function $D(p)$, decreasing in the price p. The production of the good requires the usage of technologies patented by two pure upstream innovators. Innovator $i = 1, 2$ holds a patent with strength x_i relevant for its own technology, with $x_1 \geq x_2$. Each innovator charges a per-unit royalty r_i to license the patent that covers its technology.[1] We denote the total royalty rate as $R \equiv r_1 + r_2$. We assume that there is no further cost of production so that the marginal cost of the final product is also equal to R.

Most of the results of the paper do not require that we explicitly model the pricing decision of the downstream producer in the final market. It is enough to make the following assumption on how the quantity sold depends on the aggregate royalty rate.

Assumption 1. *Define as $\tilde{D}(R)$ the total quantity sold in the final market as a function of R. This function is a decreasing and log-concave function of R, with $\tilde{D}(0) > 0$ and $\tilde{D}(R) \to 0$ as $R \to \infty$.*

These are standard regularity conditions guaranteeing that the patent holders' profit function is well-behaved. It is worth to discuss two extreme cases. When the downstream producer can extract all the surplus from consumers using perfect price discrimination, the previous assumptions imply that $D(p)$ is log-concave in p, as typically assumed in in the literature. At the other extreme, when the downstream producer chooses a unique monopoly price for the product, $\tilde{p}(R)$, this assumption imposes conditions on $\tilde{D}(R) \equiv D(\tilde{p}(R))$. Double marginalization will arise in this last case.

We denote the profits of the downstream producer as $\Pi_B(R)$. Standard arguments allow us to show that $\Pi'_B(R) = -\tilde{D}(R) < 0$ and the previous assumption implies that

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[1]As pointed out in Llobet and Padilla (2016) royalty-stacking problems are aggravated under per-unit royalties compared to the more frequent ad-valorem royalties, based on firm revenue. As discussed in section 5, the mechanisms discussed in this paper also operate when royalties are assumed to be ad-valorem but they lead to a more complicated exposition.

$\Pi_B''(R) = -\tilde{D}'(R) > 0$, so that the profits of the downstream producer are convex in R.

The royalty rate for technology i is set by innovator i as a take-it-or-leave-it offer. The downstream producer, however, might challenge in court the patent that covers that technology. Litigation involves positive costs L_B and L_U for the downstream monopolist and any upstream patent holder, respectively. The success in court of innovator i is based on the strength of its patent, x_i. In particular, the probability that a judge rules in favor of patent holder i, denoted as $g(x_i)$, is assumed to be increasing in x_i. To the extent that more substantial innovations translate into stronger patents, we can interpret the increasing function $g(x_i)$ as a reflection of this relationship.[2] If innovator i wins in court the downstream producer must pay the royalty rate r_i. Otherwise, the royalty rate of that innovator is reduced to 0. When indifferent we assume that the downstream producer prefers not to litigate.

We can summarize the payoffs of the downstream producer if it decides to only take innovator i to court as

$$(1 - g(x_i))\Pi_B(r_j) + g(x_i)\Pi_B(r_i + r_j) - L_B,$$

where r_j is the royalty rate charged by innovator $j \neq i$. If the patent of innovator j had already been litigated, profits in that case would become

$$(1 - g(x_i))\Pi_B(r_j) + g(x_i)\Pi_B(r_i + r_j) - 2L_B,$$

where the original royalty rate r_j would be replaced by 0 if the patent of innovator j had been invalidated, with probability $1 - g(x_j)$. Notice that this discussion assumes that litigation is sequential. Since royalty rates are established before any litigation takes place, the decision to simultaneously litigate both patents is equivalent to sequential litigation where the second trial occurs regardless of the initial outcome.

In the main sections of the paper we focus on the case in which L_U is relatively high so that litigation is a significant threat but it never emerges in equilibrium. That is, it is

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[2]For simplicity we abstract from situations in which upstream patent holders own the rights for technologies that might be infringed by other upstream patent holders.
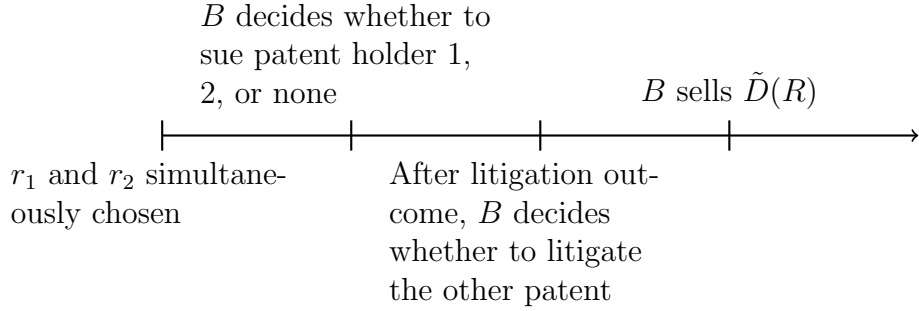
Figure 1: Timing of the model

always optimal for patent holders to choose a royalty rate that discourages litigation by the downstream producer, leading to profits of $r_i \tilde{D}(r_i + r_j)$ for innovator i given $j \neq i$. This case can be understood as a situation in which innovators individually benefit relatively less from licensing than downstream producers from avoiding to pay the royalty rate. Doing so allows us to focus on the litigation incentives of the downstream producer and compare this situation with the standard royalty-stacking case where this threat does not exist. In section 5.7 we describe the incentives for both patent holders to go to court when L_U. In that case, the expected profits of innovator i would become

$$g(x_i) r_i \tilde{D}(r_i + r_j) - L_U,$$

where r_j would be 0 if the patent of firm j had been invalidated in a previous trial. This case is developed in detail in the Online Appendix, where we discuss the conditions under which litigation arises in equilibrium and how it affects our conclusions.

The timing of the model is described in Figure 1. First, upstream innovators simultaneously choose their royalty rates. In the second stage the downstream producer chooses which patentees to take to court (if any) and the sequence. In the final stage, once litigation has been resolved, and given the outstanding royalty rate R, the downstream producer sells in the final market.

In order to characterize the equilibrium of the game depending on the strength of the patent of each firm it is useful to start with the benchmark case of a single innovator that owns both patents.

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3   A Single Innovator

Suppose that both patents are owned by the same innovator, who licenses them to the downstream producer. When litigation is not a relevant threat, the perfect complementarity between both technologies implies that any combination of royalty rates $r_1 + r_2 = R^M$, where $R^M = \arg\max_R R\tilde{D}(R)$, will maximize upstream profits.

The previous royalty rate arises in equilibrium as long as both patents are sufficiently strong — that is, if $g(x_i)$ is very close to 1 for $i = 1, 2$ — or, alternatively, when the legal costs of the downstream producer, L_B, are sufficiently high. In that case, it is not in the interest of the downstream producer to challenge in court any of the patents and it will pay the total royalty rate R^M.

Consider now the situation where one of the patents is weak. For simplicity, suppose that $g(x_1) = 1$ but $g(x_2) < 1$ so that the first patent will never be litigated by the downstream producer. This firm prefers not to challenge in court the second patent if and only if

$$(1 - g(x_2)) \left[\Pi_B(r_1) - \Pi_B(r_1 + r_2) \right] \leq L_B. \tag{1}$$

It is clear that this condition will hold if $r_1^* = R^M$ and $r_2^* = 0$. Hence, due to the perfect complementarity of the two patents, it is enough that one of them is sufficiently strong to guarantee that the monopolist innovator can attain the total royalty rate R^M without triggering litigation by the downstream producer. Of course, the optimal apportionment of the total royalty rate may not be unique if $g(x_2) > 0$ and any combination (r_1, r_2) such that $r_1 + r_2 = R^M$ that also satisfies equation (1) will yield the same profits.

Finally, suppose now that both patents are (equally) weak. To be more precise, assume that $g(x_1) = g(x_2) = g(x)$ is small and that

$$(1 - g(x)) \left[\Pi_B(0) - \Pi_B(R^M) \right] > L_B. \tag{2}$$

This condition implies that if the total monopoly royalty rate is allocated to one of the patents the downstream producer finds it worthwhile to go to court and try to invalidate

it. In this case, it is optimal for the patent holder to apportion the total royalty rate between the two patents. The highest royalty rate that the patent holder can demand for each patent is limited by the legal costs that the downstream producer must incur to challenge it in court.

Interestingly, litigation here might involve one or both patents. Sequential litigation allows the downstream producer to condition the decision to challenge a patent in court on the outcome of the previous trial. This strategy is optimal due to the convexity of $\Pi_B(R)$. This means that the litigation of a patent might be optimal when the other one has already been invalidated, but not when it has been proved to be valid. That is, it could be the case that

$$(1 - g(x)) \left[\Pi_B(0) - \Pi_B(r_i) \right] > L_B > (1 - g(x)) \left[\Pi_B(r_j) - \Pi_B(r_1 + r_2) \right],$$

for some $i = 1, 2$ and $j \neq i$. By waiting until the outcome of the first trial has been revealed, the downstream producer would benefit from the option value of going to court a second time only in those states of the world where it is worthwhile.

The next result summarizes the previous discussion and characterizes the optimal royalty rate when litigation is a relevant concern for both patents and the downstream producer chooses endogenously the sequence under which they are challenged.

Proposition 1. *When $g(x_1)(\geq g(x_2))$ is sufficiently high so that condition (2) is not satisfied, unconstrained monopoly profits can be attained. If $g(x_1) = g(x_2) = g(x)$ is sufficiently small so that condition (2) holds, then it is optimal to charge a positive royalty rate for both patents. Furthermore, when L_B is sufficiently low so that the total monopoly royalty rate, R^M, cannot be attained without triggering litigation, the optimal combination of royalty rates is unique, with $r_1^m = r_2^m = r^m$. The royalty rate is increasing in $g(x)$ and L_B, implicitly defined as*

$$g(x)\Pi_B(r^m) + (1 - g(x))\Pi_B(0) - \Pi_B(2r^m) = \frac{L_B}{1 - g(x)} + L_B. \tag{3}$$

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In order to interpret the previous result it is useful to focus first on the case in which both patents command the same royalty rate, $r_1 = r_2 = r$. Because $\Pi_B(R)$ is a convex function of R, we have that $\Pi_B(r) - \Pi_B(2r) < \Pi_B(0) - \Pi_B(r)$. This implies that if it is profitable for the downstream producer to challenge one of the patents, it will also be profitable to challenge the other one upon an initial success in court. It also means that the litigation of both patents will not be profitable if

$$(1 - g(x))\left[\Pi_B(r) - \Pi_B(2r)\right] + (1 - g(x))\left\{(1 - g(x))\left[\Pi_B(0) - \Pi_B(r)\right] - L_B\right\} \leq L_B. \quad (4)$$

The first term in the left-hand side of the equation identifies the gains of the downstream producer to challenge in court one of the patents, as described in equation (1). The second term captures the *option value* that litigation may bring about. That is, if the downstream producer wins the first trial the profitability of challenging the other patent increases. We call this result a *litigation cascade*.[3]

Because the previous expression is increasing in r and the innovator is constrained by the weak patents it owns, it is always optimal to choose the highest royalty rate compatible with not triggering the litigation of any of the patents, which determines equation (3) and, hence, r^m. Notice that the equality in equation (4) implies that challenging the first patent must yield an expected revenue lower than the cost of going to court, L_B. However, it also implies that the profits from the second trial, which occurs with probability $1 - g(x)$, compensate for the losses from going to court the first time. That is, when indifferent between going to court or not, the downstream producer is only motivated to litigate by the prospect of invalidating both patents. Of course, the previous condition also means that if the challenge to the first patent is unsuccessful, litigating the other one would be unprofitable, as the returns from that second trial would be identical to those faced for the first patent.

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[3]In practice, litigation might take years and a second trial might start before the first one has concluded if the information uncovered by the downstream producer during the process indicates that the revised probability of success is sufficiently high. The implications of such a strategy are very similar to the fully sequential setup assumed here.

When both patents are equally weak it is optimal to offer them at the same royalty rate because, by doing so, the cost of the downstream producer to try to invalidate them is maximized as, in order for litigation to be profitable it would have to incur the legal costs twice (in the second case with probability $1 - g(x)$). In contrast, charging different royalty rates could foster the litigation of the patent with the highest rate and, possibly, imply a cascade. As this would reduce the costs of the downstream producer of going to court, the monopolist patent holder would need to charge a lower total royalty rate to fend off litigation.

The three cases characterized here are a useful reference for the situation discussed next where there are two innovators. Before we do that, however, we can informally discuss the optimal royalty rate when $1 > g(x_1) > g(x_2) > 0$ and no combination of royalty rates can attain the monopoly total rate R^M without triggering litigation. By continuity, we know that when the difference between $g(x_1)$ and $g(x_2)$ is sufficiently large, patent 1 will command a higher royalty rate. The apportionment of the total rate will be such that litigating one of the patents will yield strictly negative profits and the prospect of initiating a cascade will not compensate for this loss.

4   Two Innovators

We consider now the case where each patent is owned by a different innovator. Following the previous discussion, we analyze three different situations depending on whether the two patents are strong, only one is strong, or both patents are weak.

4.1   Two Strong Patents

Suppose that both upstream innovators have a sufficiently strong patent so that litigation by the downstream producer will never be a credible threat, $g(x_1) = g(x_2) = 1$. The profits of innovator i can be defined as

$$\Pi_i(r_j) = \max_{r_i} r_i \tilde{D}(r_i + r_j), \tag{5}$$

where $j \neq i$. We denote the royalty rate that corresponds to the Nash Equilibrium of the game when firms are unconstrained by litigation as $r_i^u = r^u$ for all i. For completeness, we reproduce next the standard royalty-stacking result (see, for example, Lemley and Shapiro (2007)), which shows that this royalty rate is higher that the one we characterized in the previous section, where a unique firm maximized the profits from licensing both patents, R^M. Assumption 1 not only guarantees concavity of the patent holder's problem but it also implies that royalty rates are strategic substitutes, delivering the following result.

Proposition 2 (Royalty Stacking). *When $g(x_1) = g(x_2) = 1$ the game has a unique equilibrium in which all innovators choose $r_i^u = r^u$. The total royalty rate is higher than the one that would emerge if both patents were owned by the same innovator, $2r^u > R^M$.*

This result is a version of the Cournot-complements effect under which firms choosing quantities of complementary products induce final prices even higher than those of a monopolist. The intuition here is very similar. The decision of a patent holder to increase the royalty rate trades off the higher margin with the lower quantity sold but without internalizing the fact that this decrease in the quantity has a negative effect on the royalty revenues of the other patent holder.[4]

4.2 One Weak Patent: The Inverse Cournot Effect

Suppose now that $g(x_1) = 1$ but $g(x_2) < 1$ so that only patent holder 2 may face litigation by the downstream producer. Given the royalty rates chosen in the first stage, the downstream producer prefers not to challenge in court the patent of innovator 2 if and only if (1) holds. That is, litigation is unprofitable if the expected gains from avoiding to license the patent of innovator 2 are lower than the costs involved. The next lemma characterizes the values of r_1 for which litigation will emerge.

Lemma 3. *If $L_B > (1 - g(x_2)) [\Pi_B(0) - \Pi_B(r_2)]$ innovator 2 will not be brought to court for any $r_1 > 0$. For lower values of L_B, the downstream producer will litigate the patent*

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[4]This result holds for a generic number of firms meaning that the royalty-stacking problem becomes more severe when the total number of patents is fragmented in the hands of more firms.

of innovator 2 if $r_1 < \bar{r}_1(L_B, x_2, r_2)$. The threshold royalty rate $\bar{r}_1$ is increasing in r_2 and decreasing in L_B and x_2.

The previous lemma distinguishes two regions. When L_B is high, litigation will not be a meaningful threat. For lower values of L_B, the decision of the downstream producer to sue innovator 2 depends on the royalty rate set by the other patent holder. In particular, a positive royalty rate $r_1 < \bar{r}_1(L_B, x_2, r_2)$ chosen by innovator 1 will spur the litigation of the patent of innovator 2. The intuition is as follows. If r_1 is high, profits for the downstream producer are low, independently of whether the patent of innovator 2 is upheld in court or not. Thus, it is unlikely that the gains from litigation offset the costs involved. When r_1 is reduced, and due to the fact that the profit function $\Pi_B(R)$ is convex in R, the difference in profits when the patent of innovator 2 is upheld in court or invalidated increases, enticing downstream litigation.

An immediate consequence of this result is that if L_B is sufficiently low royalty stacking is mitigated and the equilibrium characterized in Proposition 2 will fail to exist. More interesting is the fact that, as we will see next, the threat of litigation might operate even in the case in which the original equilibrium satisfies equation (1), i.e. when $r^u > \bar{r}_1(L_B, x_2, r^u)$.

In the model without litigation, royalty stacking arises because royalty rates are strategic substitutes. Since both patent holders choose their royalty rate without anticipating that the reduction of the quantity sold downstream negatively affects the other patent holder, they engender a total royalty rate that becomes too high. The threat of litigation provides a moderating effect on the royalty rate that innovator 2 will offer to avoid being brought to court. Furthermore, innovator 1 anticipates that reducing r_1 induces a decrease of r_2. We denote this mechanism the *Inverse Cournot effect* and it operates in the opposite direction of the standard Cournot Effect.[5] This new effect generates a positive

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[5]Of course, this effect immediately generalizes to the case of N patent holders with a portfolio sufficiently strong so that it will never be litigated. In that case, the Inverse Cournot effect would indicate that the highest royalty that patentee 2 can charge is increasing in the sum of the royalty of all the other patent holders.

relationship between r_1 and r_2, allowing innovator 1 to internalize the gains that a lower royalty rate would bring about due to the higher quantity sold in the final market.

In any equilibrium with royalty rates r_1^* and r_2^*, innovator 2 will avoid being sued if (1) holds. However, this condition also implies that there will never be a Nash Equilibrium in which the downstream producer is indifferent between litigating innovator 2's patent or not. The reason is that innovator 1 would always prefer to lower a bit the royalty rate and induce innovator 2 to be brought to court. At essentially no cost, with probability $1 - g(x_2)$ innovator 1 would become the only firm licensing the technology. This deviation would be profitable as it generates a discrete increase in the quantity sold downstream. If, instead, equation (1) held with inequality, innovator 2 would find optimal to raise its royalty rate unless it were already equal to r^u. A consequence of this insight is that unless L_B is so high that the litigation threat is irrelevant and $r_1^* = r_2^* = r^u$, there will be no pure-strategy equilibrium without litigation.

Proposition 4. *An equilibrium in pure strategies and no litigation exists if and only if* $r_1^* = r_2^* = r^u$.

When L_B is small, given that demand is decreasing in the aggregate royalty rate, a Nash equilibrium in which litigation does not take place necessarily implies mixed strategies. Innovator $i = 1, 2$ randomizes according to a distribution $F_i(r_i)$ in a support $[r_i^L, r_i^H]$. Innovator 2 when choosing a lower r_2 trades off a lower probability of being sued with a higher payoff when litigation occurs and it succeeds in court. This trade-off means that innovator 2 will choose a lower expected royalty rate than when litigation was not a threat. In the case of innovator 1 two effects go in opposite directions. On the one hand, due to the Inverse Cournot effect, the innovator has incentives to lower the royalty rate r_1 in order to enjoy monopoly profits with a higher probability. On the other hand, there is a positive probability that the other patent is invalidated for a given r_1 and, in that case, it becomes optimal to increase the royalty rate.
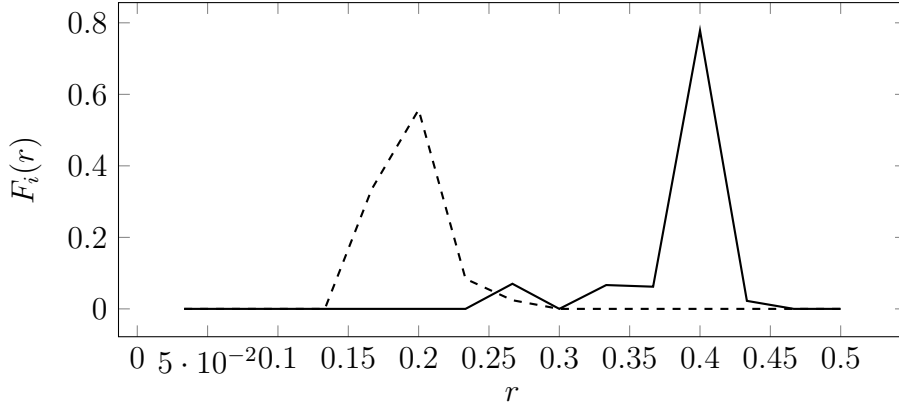
Figure 2: Mixed strategy equilibrium simulation when one of the patents is weak. The distribution of the royalty rate of innovator 1 and 2, r_1 and r_2 are indicated as a solid and dashed line, respectively. Parameters: $D(p) = 1 - p$, $g(x_2) = 1/2$, and $L_B = L_U = 1/32$.

Figure 2 provides a numerical example that illustrates the previous trade-offs. Compared to the equilibrium royalty rate when both patents are strong, $r^u = \frac{1}{3}$, innovator 2's randomization uses a lower support and will always set a lower r_2. In contrast, innovator 1 sets a higher expected royalty rate. However, the Inverse Cournot effect is reflected in the fact that the support of r_1 also includes royalty rates below r^u. As a result, the expected total royalty rate is lower in this case compared to when both patents are strong, as opposed to what occurs in the monopoly case.

The positive effect of an increase in r_1 on the royalty rate of the weak patent holder that we uncover here is new in the literature. In particular, in Choi and Gerlach (2015) downstream profits are linearly decreasing in the total royalty rate (rather than convex as in our model) and, hence, the mechanism described in Lemma 3 does not operate. In other words, in their model reducing r_1 by itself does not make suing innovator 2 more profitable. Instead, in their setup it is assumed that when the patent of innovator 2 has been successfully upheld in court the firm can raise its royalty rate. The room for an ex-post increase is lower when the royalty rate charged by innovator 1 is higher. As a result, the cost for the downstream producer of losing in court against innovator 2 decreases and it is more willing to litigate. This leads to a negative rather than a positive effect (i.e. litigation against innovator 2 becomes more attractive when r_1 goes up rather than

down, as in our model).[6]

Finally, it is useful to compare the results of our model to the case in which a monopolist patent holder sets both royalty rates at the same time. As discussed in section 3, due to the complementarity between both technologies, it is optimal for a monopolist to place most of the burden on the strong patent, for example, by choosing $r_2 = 0$ and $r_1 = R^M$. Hence, in the case of a single innovator, due to the perfect complementarity and absent any strategic effects, the total royalty rate would be unchanged as long as at least one of the patents is sufficiently strong so that litigation would never be profitable for the downstream producer. This is in contrast with the case discussed in this section, where the strategic complementarity induced by the Inverse Cournot effect provides incentives for innovators to set a total royalty rate lower than what would emerge under royalty stacking. This occurs even when litigation does not take place in equilibrium and where, as discussed earlier, a royalty rate r^u by both firms would not have made litigation worthwhile for the downstream producer.

4.3 Two Weak Patents: Strategic Effects of Litigation Cascades

We now turn to the case in which both patents are equally weak, $g(x_1) = g(x_2) = g(x) < 1$. As in the monopoly case discussed before, we assume that the downstream producer sues patent holders in an endogenous sequence that can be conditioned on the previous court outcome. Our first result characterizes the optimal order under which patents will be challenged in court.

Lemma 5. *When both patents are equally weak, it is always optimal for the downstream producer to challenge first the patent associated to the highest royalty rate.*

The higher the royalty rate of a patent holder the more likely it is that litigation pays

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[6]A positive effect arises when both patent holders are weak. In that case, when the downstream producer is indifferent between suing either of them but not both due to the litigation costs involved, the decrease in the royalty rate of innovator 1 increases the probability that the competitor is brought to court, reducing, in turn, its royalty rate. The downstream producer's upside is greater and its downside lower when suing the patent holder with the high royalty rate.

off irrespective of the outcome of the lawsuit against the other patent holder. In contrast, the litigation of the patent with the low royalty rate is less profitable and whether it is optimal to go to court or not might hinge on the outcome of the other trial. Thus, it is optimal to postpone litigation of that patent until the resolution of the first court case. In the rest of the paper we assume that when both patent holders set the same royalty rate they are brought to court first with probability $1/2$.

The previous result is useful to anticipate the changes in the probability that innovators are brought to court as a result of variations in the royalty rate. We now explore the condition under which a symmetric equilibrium may exist. As discussed in section 3, if both innovators choose the same royalty rate, the downstream producer will not be willing to litigate if (4) holds. It is immediate that this condition is less likely to be satisfied than the one driving the decision to sue innovator 2 when only this firm is constrained, as illustrated in equation (1). This comparison would suggest that, before we account for the optimal response of the innovators to the increased litigation, the royalty-stacking problem would become less severe when both patents are weak. As we will see next, the opposite may actually be true once we account for these strategic considerations.

Given r_1 and r_2 and the endogenous ordering implied by Lemma 5, we can compute the gains of the downstream producer from suing innovator 2 contingent on success in the first trial — to be compared to $L_B + (1 - g(x))L_B$ — as

$$\Phi(r_1, r_2) \equiv \begin{cases} \Pi_B(r_2) - \Pi_B(r_1 + r_2) + (1 - g(x))\left[\Pi_B(0) - \Pi_B(r_2)\right] & \text{if } r_1 > r_2, \\ \Pi_B(r) - \Pi_B(2r) + (1 - g(x))\left[\Pi_B(0) - \Pi_B(r)\right] & \text{if } r_1 = r_2 = r, \\ \Pi_B(r_1) - \Pi_B(r_1 + r_2) + (1 - g(x))\left[\Pi_B(0) - \Pi_B(r_1)\right] & \text{otherwise.} \end{cases}$$

These gross profits change with r_1 according to

$$\frac{\partial \Phi}{\partial r_1} = \begin{cases} -\Pi'_B(r_1 + r_2) & \text{if } r_1 \geq r_2, \\ \Pi'_B(r_1) - \Pi'_B(r_1 + r_2) - (1 - g(x))\Pi'_B(r_1) & \text{otherwise.} \end{cases}$$

This expression implies that increases and decreases of r_1 around r_2 have an asymmetric effect on the willingness to litigate of the downstream producer. Consider an initial situation in which $r_1 = r_2$. As expected, an increase in r_1 raises the profitability of challenging the patent of innovator 1 as the downstream profits without litigation are

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smaller. In contrast, decreases in r_1 below r_2, lead to two opposite effects. On the one hand, the first two terms correspond to the Inverse Cournot effect and imply that innovator 2 is more likely to be brought to court and, in turn, trigger a litigation cascade. On the other hand, contingent on the patent of innovator 2 being invalidated, which occurs with probability $1 - g(x)$, a lower r_1 reduces the expected gains from trying to invalidate also the patent of innovator 1 by $(1 - g(x))\Pi'_B(r_1)$. Hence, the total effect of a decrease in r_1 in the chances that innovator 1 ends up in court is in general ambiguous. The following example illustrates this point.

Example 1. *Under a linear demand function, $D(p) = 1 - p$, a downstream monopoly price, and symmetric royalty rates $r_1 = r_2 = r$, a decrease in the royalty rate of one of the innovators lowers the return from litigation of the downstream producer if and only if $r > \frac{1-g(x)}{2-g(x)}$.*

Notice that in the previous example, the unconstrained equilibrium royalty rate is $r_1^u = r_2^u = \frac{1}{3}$. Thus, if $g(x) < \frac{1}{4}$, the litigation cascade will dominate the Inverse Cournot effect, making a deviation of a patent holder from r^u unprofitable.

As opposed to the case of one weak patent, when both patents are weak the risk of a litigation cascade places a lower bound on the innovator's decrease in the royalty rate. As the next proposition states, this limit may help sustain a symmetric Nash Equilibrium in pure strategies without litigation.

Proposition 6. *Suppose that L_U is large so that there is no litigation in equilibrium. If patent holders are identical, the demand function is linear, and monopoly pricing is used, when $g(x)$ and L_B are sufficiently small, a unique symmetric equilibrium in pure strategies exists, where $r_1^* = r_2^* = r^*$ solves*

$$g(x)\Pi_B(r^*) + (1 - g(x))\Pi_B(0) - \Pi_B(2r^*) = \frac{L_B}{1 - g(x)} + L_B, \qquad (6)$$

and $r^* < r^u$. The equilibrium royalty rate is increasing in $g(x)$ and L_B.

This result provides conditions under which a pure-strategy equilibrium without litigation that differs from the one in the case of strong patents might emerge. In order to interpret this outcome, consider a deviation in the royalty rate. An increase will surely foster litigation and will not pay off when the cost L_U is high. Lowering slightly the royalty rate below r^* implies that the patent of the other innovator is litigated first. However, given that $g(x)$ is small, a litigation cascade might affect the deviating patent holder, undermining the profitability of this decision. Finally, a significant decrease in the royalty rate would discourage further litigation if the downstream producer were successful against innovator 2.[7] The lower is L_B the lower this royalty rate must be and, again, the less profitable this deviation becomes.

The comparison with the case of a single innovator allows us to highlight the implications of the litigation cascades. By Proposition 1, when patents are sufficiently weak both situations deliver the same royalty rate, $r^m = r^*$. The reason is that when patents are weak innovators are wary of a decrease in the royalty rate that might lead the other firm to court and the additional litigation that this might bring about. Consequently, the strategic considerations captured by Inverse Cournot effect become muted and firms choose in equilibrium the highest royalty rate compatible with preventing litigation by the downstream producer.[8]

The previous arguments allows us to identify the different strategic behavior when both patents are owned by different firms, compared to the case of a firm that owns both patents. In the case of a single innovator, an increase in the strength of a patent is associated with a (weakly) higher royalty rate. Interestingly, the Inverse Cournot effect suggests that this monotonicity does not need to hold when the patents are owned by two innovators and only one of them becomes stronger. In fact, when that patent becomes

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[7]This last deviation is only likely to be optimal if this decrease in the royalty rate implies a significant boost in demand. The linear-demand assumption in the text guarantees that this is not the case.

[8]Notice that the previous result does not mean that the royalty rate defined in (6) emerges under the same region of parameters in both cases. In particular, it could be that a single monopolist is not constrained by litigation and would choose R^M whereas two innovators would be forced to choose a royalty rate r^*.

sufficiently strong, the innovator is no longer constrained by the risk of being brought to court and is willing to lower the royalty rate to induce the litigation of the other patent. In the Online Appendix we provide an example where this effect results in a lower equilibrium royalty rate in spite of the stronger overall intellectual property.

Finally, although the characterization of the equilibrium in situations where both patents have an intermediate strength is difficult to establish analytically, the results from the previous cases allow us to obtain a few insights. The equilibrium will be, in general, in mixed strategies with a support of the royalty rate of each firm increasing in the strength of its own patent portfolio. The effect of the strength of the portfolio of the other patent holder, however, would be ambiguous. On the one hand, a weaker patent makes the Inverse Cournot effect more relevant, driving down the royalty rate. On the other, if the patent of the other firm turns out to be very weak, the risk of a litigation cascade discourages the choice a low royalty rate.

5   Robustness and Extensions

We now study the effect of changing some of the maintained assumptions throughout the paper.

5.1   Ad-Valorem Royalties

Although most of the literature on innovation has assumed that royalties are paid per unit sold in the downstream market, in many technological industries patents are licensed using ad-valorem royalties, understood as a percentage of the revenue of the licensee.[9] As Llobet and Padilla (2016) show, absent litigation, ad-valorem royalties mitigate the royalty stacking problem.

In this section we show that the same moderating force introduced by the Inverse Cournot effect also arises under ad-valorem royalties. In particular, consider the case in

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[9]See, for example, Bousquet et al. (1998). As discussed in section 5.6, royalties are also combined with a fixed fee.

which the downstream producer faces a demand $D(p)$ and it incurs in a marginal cost of production $c > 0$.[10] When innovators 1 and 2 charge ad-valorem royalties s_1 and s_2 and the aggregate rate is $S \equiv s_1 + s_2$, the problem of the downstream producer that chooses a final price p can be written as

$$\Pi_B(S) = \max_p \left[(1 - S)p - c\right] D(p).$$

The resulting monopoly price, p^M, is increasing in S under standard regularity conditions, such as the log-concavity of the demand function. This requirement is also enough to show that $\Pi_B(S)$ is decreasing and convex in S. As a result, if we consider the case in which $g(x_2) < g(x_1) = 1$, the downstream producer will challenge in court the patent of innovator 2 if s_1 is lower than a threshold level $\bar{s}_1$, defined as

$$(1 - g(x_2)) \left[\Pi_B(\bar{s}_1) - \Pi_B(\bar{s}_1 + s_2)\right] = L_B. \tag{7}$$

It is immediate that a counterpart of Lemma 3 can be obtained in this case, with $\bar{s}_1$ increasing in s_2. Patent holder 1 has incentives to lower s_1 in order to induce patentee 2 to lower s_2 and prevent being litigated. See the Online Appendix for an example.

5.2 Downstream Competition

In this section we show that as downstream competition increases, the Inverse Cournot Effect is moderated but it does not necessarily disappear. There are two reasons for this weaker effect. First, more competition leads not only to lower downstream profits but also to lower differential profits from invalidating a patent. Second, a free-riding problem arises. If courts invalidate the patent of one of the innovators the royalty rate that all downstream producers pay to that firm is also reduced to 0.

Regarding the first effect, consider a downstream market with N identical competitors. Their only marginal cost of production is the total royalty rate R. Denote profits as

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[10]As it is well-known, the problem when $c = 0$ is trivial, since an ad-valorem royalty rate of 100% would always be optimal, as it would create no distortion in the final market.

$\Pi_B(R, N)$. Under standard conditions, $\Pi_B(R, N)$ is decreasing in both arguments and convex in R.

Consider the case where $g(x_1) = 1$. Suppose that if a total of $n \leq N$ downstream firms challenge the patent of innovator 2 in court, it will be considered valid with probability $g(x_2, n)$, weakly decreasing in n. Each downstream producer incurs a litigation cost L_B by going to court.

Any downstream firm will be indifferent between challenging patent 2 or not, assuming that no other downstream firm goes to court too, if $r_1 \leq \bar{r}_1$, defined as

$$(1 - g(x_2, 1)) \left[\Pi_B(\bar{r}_1, N) - \Pi_B(\bar{r}_1 + r_2, N) \right] = L_B.$$

As in the baseline model, the Inverse Cournot effect arises due to the convexity of the profit function with respect to R. Furthermore, if $\frac{\partial \Pi_B}{\partial R \partial N} > 0$ then $\frac{d\bar{r}_1}{dN} < 0$. This condition holds under many of the typical demand specifications.

Example 2 (Cournot Competition). *Under a linear demand function $P(Q) = a - Q$, where $R < a$, $\frac{\partial \Pi_B}{\partial R \partial N} = \frac{a-R}{2N} > 0$. When demand is isoelastic, $P(Q) = Q^{-\frac{1}{\eta}}$, the cross derivative of the equilibrium profit function corresponds to*

$$\frac{\partial \Pi_B}{\partial R \partial N} = (\eta - 1)\eta^{-\eta} R^{-\eta} (\eta - N)^{\eta - 2} > 0.$$

Example 3 (Product Differentiation). *Suppose that downstream producers sell differentiated products, with a degree of substitubility identified by the parameter $\gamma \geq 0$. Firm i faces a demand function*

$$q_i = \frac{1}{N} \left[v - p_i(1 + \gamma) + \frac{\gamma}{N} \sum_{j=1}^{N} p_j \right].$$

Using the expression for the profits in the symmetric equilibrium we have

$$\frac{\partial \Pi_B}{\partial R \partial N} = \frac{2(v - R)\left[(N - 1)\gamma(3 + \gamma) + 2N\right]}{((N - 1)\gamma + 2N)^3} > 0.$$

In the previous examples as the number of downstream firms increases the Inverse Cournot effect becomes weaker.[11] That is, innovator 1 must decrease the royalty rate

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[11]An exception is the circular city, where the inelastic demand implies that the cross-derivative is 0 and, thus, the Inverse Cournot effect is independent of the number of firms.

further in order to induce litigation against the other innovator. The second example indicates that as product differentiation increases, understood as a decrease in γ, the threshold value $\bar{r}_1$ increases. Product differentiation is akin to a decrease in competition.

Finally, in order to study the free-riding effect, let's focus on the case with $N = 2$. The profits of a downstream producer when $n \leq 2$ firms challenge the patent in court, gross of litigation costs, can be written as

$$V_B(n) = (1 - g(x_2, n))\Pi_B(r_1, 2) + g(x_2, n)\Pi_B(r_1 + r_2, 2).$$

Suppose that it is worthwhile for the two downstream firms to challenge in court patent 2. That is, $2V_B(2) - 2L_B > 2V_B(1) - L_B$. It is easy to see that if one of the firms litigates, the other firm will also litigate if and only if $V_B(2) - L_B > V_B(1)$. As a result, if $V_B(2) - V_B(1) < L_B < 2[V_B(2) - V_B(1)]$ litigation that would increase the overall value for downstream firms will not take place, due to the lack of coordination.

5.3   Royalty Renegotiation

The timing of the model assumes that once patent holder i chooses the royalty rate r_i the downstream producer will end up paying that amount unless it is brought to court and the patent invalidated. This means that when the patent is considered valid by the court its owner has no chance to increase the royalty rate. Papers like Choi and Gerlach (2015) allow for the possibility of renegotiation under these new circumstances. As we discuss next, royalty renegotiation weakens the Inverse Cournot effect but it does not qualitatively affect the results of the paper.

In the benchmark model, the maximum royalty rate that innovator 1 could set and induce litigation on the patent of innovator 2, $\bar{r}_1$, can be obtained by setting (1) with equality. Under royalty renegotiation, $\bar{r}_1$ would now arise from

$$(1 - g(x_2))\left[\Pi_B(\bar{r}_1) - \Pi_B(\bar{r}_1 + r_2)\right] + g(x_2)\left[\Pi_B(r^M) - \Pi_B(\bar{r}_1 + r_2)\right] = L_B,$$

where, after the success of the downstream producer against innovator 2, the royalty rate increases to the monopoly one, r^M. In this case it is still true that the Inverse

Cournot effect operates, since $\bar{r}_1$ increases in r_2, but only when the patent of innovator 2 is sufficiently weak. This observation is in opposition to the results in Choi and Gerlach (2015), where the positive relationship between the royalty rate of both firms is generated precisely by the upside that royalty renegotiation provides.

5.4 FRAND Licensing

Most SSOs request participating firms to license the patents that are considered essential to the standard according to Fair, Reasonable, and Non-discriminatory (FRAND) terms. The ambiguity of this term and the different interpretation of patent holders and licensees has made FRAND a legally contentious issue. Courts have sometimes been asked to determine whether a royalty rate is FRAND or not and in some instances to set the FRAND rate.

The goal of this section is not to assert whether a royalty rate is FRAND or not but, rather, to study what is the effect of courts determining it on the previous results and, in particular, on the Inverse Cournot effect. In order to do so, we now extend the basic setup and assume that when a patent is valid, the downstream producer can ask the court to rule that it is essential to the standard and the royalty requested is not FRAND. We assume that the stronger is a patent the more likely it is that the technology it covers is considered essential to the standard. This probability is defined as $h(x_i)$, increasing in x_i. The arguments apply to the existence of N patent holders, with R_{-i} corresponding to the sum of the royalty rate of all patentees other than i.

If the patent is declared to be essential to the standard the court will determine the appropriate royalty rate, $\rho(x_i, r_i, R_{-i})$. We assume that this rate is an increasing function of the strength of the patent, x_i. As we discuss later, we also allow for the possibility that the court's decision depends on the royalty announced by the patent holder or the total royalty established by the other patent holders.

Following the analysis in the benchmark model, the downstream monopolist will be

interested in challenging patent i in court only if

$$(1 - g(x_i)) \left[\Pi_B(R_{-i}) - \Pi_B(R_{-i} + r_i)\right]$$

$$+ g(x_i)h(x_i) \left[\Pi_B(R_{-i} + \rho(x_i, r_i, R_{-i})) - \Pi_B(R_{-i} + r_i)\right] > L_B.$$

The previous expression has a straightforward interpretation. The downstream producer might benefit from litigation either because the patent is ruled invalid, which occurs with probability $1 - g(x_i)$, or because it is considered valid and essential to the standard, with probability $g(x_i)h(x_i)$. In this latter case, the royalty rate drops from r_i to $\rho(x_i, r_i, R_{-i})$.

Lemma 7. *Suppose $\rho(x_i, r_i, R_{-i})$ is independent of r_i and R_{-i}, $\rho(x_i)$. Then, there exists a unique critical value $\bar{r}_i(x_i, R_{-i}, L_B)$ such that the producer prefers to challenge in court patent i if and only if $r_i > \bar{r}_i$. Furthermore, this threshold is increasing in R_{-i} and L_B.*

This result indicates that the Inverse Cournot effect is qualitatively unaffected as long as the court determines the FRAND royalty only as a function of the strength of the patent. The main difference, however, is that the result does not guarantee that innovators with a stronger patent can indeed charge a higher royalty without triggering litigation by the downstream producer. Although a higher x_i reduces the probability that the court invalidates the patent, it also increases the probability that the patent is considered essential and, thus, that the royalty rate is diminished from r_i to $\rho(x_i)$. This second effect prevails when increases in x_i have a large impact on $h(x_i)$ but a small one on $\rho(x_i)$.

The previous lemma establishes sufficient conditions and the result might still hold even if, as it is plausible, $\rho(x_i, r_i, R_{-i})$ increases in r_i. An interesting case that it is worth to mention is the following: Suppose that a court would determine the FRAND royalty rate as a function of x_i but it would never choose $\rho(x_i, r_i, R_{-i})$ higher than r_i. It can be shown that the results are preserved in that case.

Finally, there have been instances in which courts have used existing licensing agreements in order to pin down the FRAND royalty rate for a patent (or patent portfolio).

Interestingly, they have been used in two directions. In some cases, courts have adopted the so-called *comparables approach* and set the royalty rate according to the rate negotiated for comparable patents, even in the same standard.[12] In those cases increases in R_{-i} have a positive effect on $\rho(x_i, r_i, R_{-i})$ and strengthen the Inverse Cournot effect.

In other cases, and more specifically in the Microsoft v. Motorola case,[13] it has been argued that the FRAND royalty rate of a patent holder should be lowered due to the already large royalty stack. This reasoning would make $\rho(x_i, r_i, R_{-i})$ non-increasing in R_{-i}. Interestingly, this result would undermine the Inverse Cournot effect and it might even reverse its sign, with self-defeating consequences. Large patent holders would anticipate that by choosing a higher royalty rate, weaker competitors facing litigation would be forced by the court to set a lower rate, exacerbating the royalty-stacking problem that courts were aiming to address in the first place.

5.5 Sequential Royalty Setting

In the benchmark model firms choose their royalty rates simultaneously. Let's consider now the case in which the innovator with the strong patent, innovator 1, decides the royalty rate first. Innovator 2 is constrained by litigation due to its weak patent and chooses later. It is easy to see that the main forces at play in the simultaneous case will apply here. The decision of patent holder 1 in both cases would internalize the effect of r_1 on the incentives of patent holder 2 to lower r_2 and prevent litigation. Of course, in opposition to the simultaneous move case, here a pure strategy equilibrium will exist.

The similarity between the simultaneous and the sequential move game is useful to explain the behavior of large innovators that participate in SSOs. These firms devote substantial resources in developing technologies the profitability of which depends on the success in the final-good market of the products that embed them. The announcement of a low royalty rate early in the standardization process can, thus, be understood as a

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[12]See Leonard and Lopez (2014) for a discussion of this and other approaches used to determine FRAND royalty rates.

[13]Microsoft Corp v. Morotola Inc, 854 F. Supp 2d 933 - Dist Court WD Washington 2012.

commitment that the royalty rate of complementary technologies developed by firms with a weaker patent portfolio would also be low, reducing the risk of royalty stacking. This interpretation is consistent with the adoption of some standards in recent years. For example, the main sponsors of the fourth-generation mobile telecommunications technology announced the licensing condition for their (essential) patents very early in the process.[14]

5.6 Two-Part Contracts

As widely assumed in the literature, innovators in the benchmark model can only use per-unit royalty rates. In this section we explore the implications of enlarging the kind of contracts that patent holders can use to accommodate two-part tariffs, combining royalty rates and fixed fees. These contracts have been documented, for example, in the biomedical industry (Hegde, 2014).

Consider the case of two patent holders $i = 1, 2$ that own one patent and offer a two-part tariff (r_i, F_i), where r_i and F_i are the royalty rate and the fixed fee, respectively. Following the reduced-form specification proposed by Calzolari et al. (2020), we assume that a fixed fee $F_i \geq 0$ generates a distortion in the final market of $\mu \geq 0$. This parameter aims to capture some of the reasons that the literature has proposed to reconcile the fact that royalties are used in spite of the double-marginalization they generate. It has been argued that two-part contracts are optimal, for example, as part of a risk-sharing strategy (Bousquet et al., 1998) or in situations where there is downstream competition (Hernández-Murillo and Llobet (2006), Reisinger and Tarantino (2019)).

The next lemma summarizes the results when patents are ironclad, $g(x_1) = g(x_2) = 1$, and they are owned by different firms. They choose simultaneously the contract to be offered to the downstream producer, (r_i, F_i) for $i = 1, 2$. As in the benchmark model,

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[14]The mechanism used to spur the adoption of a new technology that this paper uncovers resembles contractual arrangements that we observe in other technological contexts. See Gambardella and Hall (2006) for a study of the public-good problem faced in software development when placed in the public domain. In the case of encryption technologies the risk that non-practicing entities might try and enforce their patents has encouraged agents more invested in the development of software to make it open source and royalty free. See "A rush to patent the blockchain is a sign of the technology's promise" (2017, 14 January), The Economist (downloaded on 8 February 2017).

for any $\mu > 0$ the total royalty rate will be higher than in the single innovator case. As it is usually the case, we assume that fees must be non-negative.[15] When $\mu = 0$ it is immediate that $r_1 = r_2 = 0$ is always optimal and no distortion arises.

Lemma 8. *When both innovators own ironclad patents and offer two-part tariffs, their royalty rate in a symmetric equilibrium, $r^u(\mu)$, leads to royalty stacking when $\mu > 0$. That is, $R^u(\mu) \equiv 2r^u(\mu) > R^M(\mu)$ for all $\mu > 0$, where $R^M(\mu)$ is the rate chosen by a single innovator. Royalty rates are increasing in μ. When $\mu = 0$, $r^u(0) = R^M(0) = 0$ and the royalty rate maximizes social welfare, regardless of the number of innovators.*

Notice that, as expected, when fixed fees generate higher distortions, the royalty rate that innovators choose in equilibrium increases. In the limit, when μ tends to infinity the equilibrium royalty rates coincides with those in Proposition 2. Royalty stacking occurs for all positive values of μ.

Consider now the case where one patent is strong and the other one is weak, $g(x_2) < g(x_1) = 1$. As in the benchmark model, a single innovator will still be able to choose a total royalty rate that coincides with the case in which both patents are strong.

When the patents are owned by different firms, however, we now show that innovator 1 will have incentives to deviate and lower the royalty rate if the other patent is sufficiently weak, in line with the results in the rest of this paper. The condition that establishes that it is in the interest of the downstream producer to challenge patent 2 in court is

$$(1 - g(x_2)) \left[\Pi_B(r_1) - \Pi_B(r_1 + r_2) + (1 + \mu)F_2 \right] > L_B, \tag{8}$$

where, $\Pi_B(R)$ denotes the profits of the downstream firm gross of fixed fees. This condition indicates that, similarly to what happens in the benchmark model with pure royalties, the lower is r_1 the lower should be r_2 and/or F_2 to guarantee that litigation is not profitable. Notice that the fixed fee F_1 is irrelevant for this decision, since it enters the profit function of the downstream producer linearly. Of course, F_1 affects the optimal level of

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[15]Negative fees would attract potential licensees that have no intention to produce.

F_2 as it must guarantee that the downstream producer is willing to participate. That is, in an equilibrium without litigation, it must be that $\Pi_B(r_1 + r_2) - (1 + \mu)(F_1 + F_2) \geq 0$.

The next result shows that if a pure strategy equilibrium exists it must lead to the royalty-stacking outcome described above.

Proposition 9. *When $g(x_1) = 1 > g(x_2)$ an equilibrium in pure strategies without litigation and two-part tariffs exists if both firms set the royalty rate $r^u(\mu)$. This equilibrium only exists when x_2 is sufficiently high.*

The previous result is the counterpart of Proposition 4 for the case of two-part tariffs. It implies that if condition (8) holds when both firms charge a royalty rate r^u and $F_2 = 0$, a pure strategy equilibrium will fail to exist. This is likely to be the case when x_2 is low and it implies that the Inverse Cournot effect will re-emerge with similar implications to those highlighted in the baseline model. Innovator 1 would have incentives to lower the royalty rate below $r^u(\mu)$ in order to foster litigation against innovator 2. Notice that in that case, the innovator can benefit from the invalidation of the patent of the rival not only because of the increase in the quantity it can bring about but also due to the higher fixed fee that the downstream producer will be willing accept.

5.7 Equilibrium Litigation

Throughout the paper we have assumed that although litigation is a relevant threat it does not arise in equilibrium. This assumption is consistent with a high legal cost for patent holders, L_U.

In the Online Appendix we develop the situation where L_U is low and one of the patents is weak. In that case, the weak patent holder faces two possibilities. It can either lower the royalty rate, as it has been assumed throughout the paper, and avoid litigation or charge a higher royalty rate and risk having the patent invalidated and incurring in the corresponding legal costs. This second possibility is more likely to be optimal when r_1 is low. This is due to two reasons. First, the Inverse Cournot effect implies that avoiding

litigation requires a low r_2. Second, as with the standard Cournot effect, royalty rates are strategic substitutes when litigation takes place in equilibrium and, therefore, in that case it is optimal to charge a higher royalty rate. Both effects combined imply that avoiding litigation requires a large decrease in the royalty rate when r_1 is low, leading to a large reduction in revenues.

The previous result places a limit on the Inverse Cournot effect. A very low r_1 might not become profitable for the strong patent holder if it implies a large r_2 that engenders litigation and it results on a high royalty stack with probability $g(x_2)$.

6   Concluding Remarks and Policy Implications

The existence of royalty stacking in the context of technology licensing has been argued in analogy with the classical case of Cournot complements. This paper shows, however, that these insights do not necessarily carry through when we explicitly consider patent litigation and, most specifically, the incentives that firms have to make strategic use of it.

The implications of reconsidering the idea of royalty stacking through the lens of a model of patent litigation are far-reaching. One of the main contexts in which these changes apply is in the case of Standard Setting Organizations. Royalty stacking has been used to assess the desirability of patent consolidation or disaggregation. The concern about "privateers" — spin-offs of existing firms aimed at enforcing their intellectual property — and "patent assertion entities" is that they can be used to increase the royalty stack. In contrast, consolidation efforts through patent acquisitions or the creation of patent pools have been encouraged as they would contribute to lower the aggregate royalty rate.[16]

Our model suggests that these rules should be implemented with caution and that the

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[16]See Lerner and Tirole (2004). Other papers, however, have pointed out that patent pools might reduce social welfare when they include non-essential patents (Quint, 2014) or when some licensors and producers are vertically integrated (Reisinger and Tarantino, 2019).

impact of the litigation threat should be factored in. Since litigation decisions depend on the strength of the patent portfolio, if patentees pool their patents they are likely to make enforcement more effective. This last effect implies that the formation of a patent pool or the merger of patent holders might make the royalty-stacking problem worse, particularly if not all patent holders are included and the portfolios become more similar in strength.[17] By the same token, to the extent that patent holders decide to disaggregate their patent portfolio into more asymmetric patent holdings, the outcome could be socially beneficial. To evaluate the impact of these decisions we should account for how the moderation force of large patent holders that the Inverse Cournot effect brings about is mitigated or strengthened.

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[17]In our model, a patent pool including all firms will always eliminate the royalty stack and increase overall profits. Of course, to the extent that the Inverse Cournot effect reduces the size of this royalty stack, the incentives to form a pool are diminished.

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A Proofs

The main results of the paper are proved here.

Proof of Proposition 1: Following the discussion in the text, if $g(x_1)$ is such that condition (2) is not satisfied, $r_1^* = R^M$ and $r_2^* = 0$ allows the innovator to attain monopoly profits.

When $g(x_1) = g(x_2) = g(x)$ is sufficiently small so that condition (2) is satisfied, it is immediate that a necessary condition to attain the monopoly total royalty rate R^M is that the innovator chooses $r_1 > 0$ and $r_2 > 0$.

The single innovator maximizes the following function

$$\max_{r_1,r_2} (r_1 + r_2)\tilde{D}(r_1 + r_2)$$

s.t $(1 - g(x))\{\Pi_B(r_i) - \Pi_B(r_1 + r_2) + \max\{(1 - g(x))[\Pi_B(0) - \Pi_B(r_i)] - L_B, 0\} \leq L_B$

for $i = 1, 2$.

Suppose towards a contradiction that the two royalty rates that maximize profits are different and, without loss of generality, $r_1^m > r_2^m$. It is immediate that in that case it cannot be that the initial litigation of both of the patents leads to a cascade or none of them does. For the former, suppose that both of them lead to a cascade. The two constraints can be written in this case as

$$\Pi_B(r_1^m + r_2^m) \geq g(x)\Pi_B(r_i^m) + (1 - g(x))\Pi_B(0) - L_B - \frac{L_B}{1 - g(x)}.$$

Since $\Pi_B(r)$ is decreasing in r the constraint for litigating first patent 2 is binding. Hence, profits could be increased by diminishing r_1 and increasing r_2. A similar argument applies when there is no litigation cascade.

Hence, if different royalty rates are optimal, it has to be that a cascade only occurs in one of the cases. Furthermore, the cascade must occur only after success against patent 2 and only when

$$\Pi_B(0) - \Pi_B(r_1^m) > \frac{L_B}{1 - g(x)} \geq \Pi_B(0) - \Pi_B(r_2^m). \tag{9}$$

Hence, the optimal royalty rates must satisfy the constraints,

$$\Pi_B(r_1^m + r_2^m) = g(x)\Pi_B(r_1^m) + (1 - g(x))\Pi_B(0) - L_B - \frac{L_B}{1 - g(x)}, \tag{10}$$

$$\Pi_B(r_1^m + r_2^m) = \Pi_B(r_2^m) - \frac{L_B}{1 - g(x)}. \tag{11}$$

This cannot occur. In particular, combining (10) and (11) we have that

$$g(x)\Pi_B(r_1^m) + (1 - g(x))\Pi_B(0) - L_B = \Pi_B(r_2^m) \geq \Pi_B(0) - \frac{L_B}{1 - g(x)},$$

where the last inequality arises from (9). This implies that $\Pi_B(0) - \Pi_B(r_1^m) \leq \frac{L_B}{1-g(x)}$, which is a contradiction with the condition that litigating patent 1 after success in invalidating patent 2 is profitable. Hence $r_1^m = r_2^m = r^m$.

Notice that $\Pi_B(0) - \Pi_B(r^m) > \frac{L_B}{1-g(x)}$ so that a second trial would take place after an initial success. A royalty rate that did not lead to a cascade would be lower and, therefore, yield lower profits.

Finally, since the left-hand side of equation (3) is increasing in r^m it is immediate, using the Implicit Function Theorem, that this royalty rate is increasing in L_B and $g(x)$. $\qquad \square$

Proof of Proposition 2: The optimal royalty of patentee i resulting from (5) is determined using the first-order condition

$$\tilde{D}(R) + r_i^u \tilde{D}'(R) = 0. \implies r_i^u = -\frac{\tilde{D}(R)}{\tilde{D}'(R)}.$$

Replacing $r_i^u = r^* = \frac{R^u}{N}$ where $N \le 2$ is the number of firms, we can use the Implicit Function Theorem to compute

$$\frac{dR^u}{dN} = \frac{\frac{R^u}{N}\tilde{D}'(R^u)}{\tilde{D}'(R^u) + \frac{R^*}{N}\tilde{D}'(R^u) + \frac{1}{N}\tilde{D}'(R^u)} > 0.$$

The last inequality arises from a negative numerator due to $\tilde{D}'(R) < 0$ and a negative denominator that it is also negative due to the quasiconcavity of $\tilde{D}(R)$. In particular, this result implies that $R^M = R^u(1) < R^u(2) = 2r^u$. $\qquad\square$

Proof of Lemma 3: Define $\bar{r}_1$ as the value of r_1 for which equation (1) is satisfied with equality. First, we establish that it is unique and well-defined for all positive values of L_B. The left-hand side of that equation is always decreasing in r_1 for $r_2 > 0$. Furthermore, as $\tilde{D}(R) \to 0$ when $R \to \infty$ we have that the left-hand side expression can be arbitrarily small as r_1 increases. When $L_B < (1 - g(x_2))(\Pi_B(0) - \Pi_B(r_2))$ the threshold value $\bar{r}_1$ is always positive.

Using the fact that $\Pi_B'(R) < 0$ and $\Pi_B''(R) > 0$, we can compute, for $r_2 > 0$,

$$\frac{d\bar{r}_1}{dL_B} = \frac{1}{\Pi_B'(\bar{r}_1) - \Pi_B'(\bar{r}_1 + r_2)} < 0,$$
$$\frac{d\bar{r}_1}{dx_2} = \frac{g'(x_2)\left[\Pi_B(\bar{r}_1) - \Pi_B(\bar{r}_1 + r_2)\right]}{\left[\Pi_B'(\bar{r}_1) - \Pi_B'(\bar{r}_1 + r_2)\right]} < 0,$$
$$\frac{d\bar{r}_1}{dr_2} = \frac{\Pi_B'(\bar{r}_1 + r_2)}{\Pi_B'(\bar{r}_1) - \Pi_B'(\bar{r}_1 + r_2)} > 0.$$

$\qquad\square$

Proof of Proposition 4: Following the arguments in the text, suppose towards a contradiction that an equilibrium without litigation (r_1^*, r_2^*) exists with $r_1^* > 0$ and $r_2^* > 0$, different from the unconstrained solution, r^u, as defined in Proposition 2. Since $r_2^* > 0$, this equilibrium must satisfy equation (1). Suppose that this condition is satisfied with strict equality. It is easy to see that in that case r_1^* would not be optimal for patent holder 1, as it could be slightly diminished, leading to a discrete increase in final market sales from $\tilde{D}(r_1^* + r_2^*)$ to almost $g(x_2)\tilde{D}(r_1^* + r_2^*) + (1 - g(x_2))\tilde{D}(r_1^*)$.

Hence, condition (1) must be satisfied with strict inequality in the equilibrium. This means that patent holder 1 chooses the royalty as the result of $r_1^* = \arg\max r_1\tilde{D}(r_1 + r_2^*)$. Since the equilibrium differs from the unconstrained one and condition (1) constitutes an upper bound for the royalty rate of patent holder 2, it has to be the case that r_2^* is lower than the best response to r_1^*. But this is a contradiction, since patent holder 2 could always increase the royalty rate while the constraint still holds.

Alternatively, suppose that at least one of the royalty rates is 0 in equilibrium. This would be a contradiction, since both patent holders could guarantee positive profits and avoid litigation by choosing a positive royalty rate, unless $\tilde{D}(r_1^* + r_2^*) = 0$. But in that case the patent holder with a positive royalty rate would obtain higher (and positive) profits by decreasing it. □

Proof of Lemma 5: Suppose without loss of generality that $r_1 > r_2$. The optimal policy of the downstream producer can be described as arising from the following two stages. In the first stage, it decides whether to sue patent holder 1 or 2 or none at all. Upon observing the outcome of the first trial the downstream producer decides whether to sue the other patent holder or not.

Suppose that in the first stage patentee i was brought to court. Then, if it is optimal for the downstream producer to sue patentee j upon defeat it is also optimal to litigate upon success since, by convexity of $\Pi_B(R)$,

$$\Pi_B(r_i) - \Pi_B(r_i + r_j) \leq \Pi_B(0) - \Pi_B(r_j),$$

for $i = 1, 2$ and $j \neq i$. Furthermore, notice that

$$\Pi_B(r_1) - \Pi_B(r_1 + r_2) \leq \Pi_B(r_2) - \Pi_B(r_1 + r_2),$$
$$\Pi_B(0) - \Pi_B(r_2) \leq \Pi_B(0) - \Pi_B(r_1).$$

Hence, two possible orderings can arise depending on whether $\Pi_B(r_2) - \Pi_B(r_1 + r_2)$ is higher or lower than $\Pi_B(0) - \Pi_B(r_2)$. In order to determine the profits of the downstream producer in each case, we need to see how these profits compare with $\frac{L_B}{1-g(x)}$.

(i) Suppose that when 1 is sued first it is always optimal to sue 2 afterwards. Obviously, if the opposite order yields the same order, both options are equivalent and profits are identical.

(ii) Suppose that when 1 is sued first it is only optimal to sue 2 after victory. This implies that $\Pi_B(r_1) - \Pi_B(r_1 + r_2) < \frac{L_B}{1-g(x)} \leq \Pi_B(0) - \Pi_B(r_2)$. Profits become

$$g(x) \left[\Pi_B(r_1 + r_2) - L_B\right] + (1 - g(x)) \left[g(x)\Pi_B(r_2) + (1 - g(x))\Pi_B(0)\right] - L_B,$$

which, by definition, are higher than those that arise in the first case. If after litigating the portfolio of patent holder 2 it is then optimal to litigate the portfolio of the other patent holder always, this option would be, therefore, dominated by (i).

Alternatively, it could be that when patent holder 2 is sued first it is only optimal to sue patent holder 1 upon victory. Profits in that case would be

$$g(x) \left[\Pi_B(r_1 + r_2) - L_B \right] + (1 - g(x)) \left[g(x) \Pi_B(r_1) + (1 - g(x)) \Pi_B(0) \right] - L_B,$$

which are lower than when patent holder 1 is sued first.

(iii) Suppose that when patent holder 1 is sued first it is never optimal to litigate the portfolio of patent holder 2 afterwards. Profits would be

$$g(x) \Pi_B(r_1 + r_2) + (1 - g(x)) \Pi_B(r_2) - L_B.$$

If it is always optimal to sue patent holder 1 after patent holder 2 has been sued first, these profits are lower because, as in the previous case, they coincide with the profits in the first option. If instead it was optimal to litigate only upon success, again, these profits are dominated by the second option. Finally, if it is never optimal to sue patent holder 1, profits become

$$g(x) \Pi_B(r_1 + r_2) + (1 - g(x)) \Pi_B(r_1) - L_B,$$

which are again lower.

(iv) Using the same argument, if $\frac{L_B}{1 - g(x)}$ is sufficiently high so that it is never optimal to sue patent holder 1 only, bringing to court patent holder 2 only must also be dominated.

$\square$

Proof of Proposition 6: Consider a symmetric equilibrium in which patent holder 1 and 2 are constrained. This implies that $\Phi(r^*, r^*) = \frac{L_B}{1 - g(x)} + L_B$. Each firm obtains profits $r^* \tilde{D}(2r^*)$. It is immediate that r^* is increasing in L_B and $g(x)$.

Three possible deviations of a patent holder, say patentee 1, should be considered:

(i) Patentee 1 might increase its royalty to $r_1 > r^*$. This firm will be litigated first and profits, defined as $\max_{r_1} g(x) r_1 \tilde{D}(r_1 + r^*) - L_U$, will be lower if L_U is sufficiently high.

(ii) Patentee 1 might deviate by lowering the royalty rate slightly. In this case, the sign of $\frac{\partial \Phi}{\partial r_1}$ becomes relevant. In particular,

$$\frac{\partial \Phi}{\partial r_1}(r_1, r_2) \geq 0 \iff g(x) \Pi_B'(r_1) - \Pi_B'(r_1 + r_2) = \tilde{D}(r_1 + r_2) - g(x) \tilde{D}(r_1) \geq 0,$$

If $\frac{\partial \Phi}{\partial r_1} \geq 0$, decreases in r_1 reduce the incentives for the downstream firm to litigate. Since royalties are strategic substitutes and r^* is below the unconstrained royalty this strategy can never be optimal.

Alternatively, if $\frac{\partial \Phi}{\partial r_1} < 0$, a deviation consisting in a slight decrease in r_1 induces litigation, first against patentee 2 and, upon success, against patentee 1. This implies that the profits of patentee 1 become

$$g(x)r^* D(2r^*) + (1 - g(x)) \left[g(x)r^* D(r^*) - L_U \right],$$

This deviation is unprofitable if

$$L_U > r^* D(p^M(2r^*)) - g(x)r^* D(p^M(r^*)),$$

which holds given that the right-hand side is negative when $\frac{\partial \Phi}{\partial r_1}(r^*, r^*) < 0$.

(iii) Finally, patent holder 1 could lower r_1 enough so that $(1 - g(x)) \left[\Pi_B(0) - \Pi_B(r_1) \right] \leq L_B$. In that case, patent holder 1 would not be brought to court. Again, two possibilities can arise here depending on whether the downstream producer is interested in suing patentee 2 or not. Notice that only if patentee 2 is sued this deviation might be profitable. Hence, the optimal deviation is $\tilde{r}_1 = \min\{r_1^A, r_1^B\}$, where the values r_1^A and r_1^B are defined as

$$(1 - g(x)) \left[\Pi_B(0) - \Pi_B(r_1^A) \right] = L_B,$$
$$(1 - g(x)) \left[\Pi_B(r_1^B) - \Pi_B(r^* + r_1^B) \right] = L_B.$$

When r^* is sufficiently high the first constraint will be binding. Profits in either case will be $g(x)r_1 \tilde{D}(r^* + \tilde{r}_1) + (1 - g(x))r_1 \tilde{D}(\tilde{r}_1)$.

When $g(x)$ is sufficiently small it is clear that the first deviation is always dominated since it would imply profits of $-L_U$. The second deviation is also unprofitable since when $g(x) = 0$, $\frac{\partial \Phi}{\partial r_1} \geq 0$.

Regarding the last deviation, we know that $\tilde{r}_1 \leq r_1^B$. Under a linear demand when $g(x) = 0$ and monopoly pricing, we have that $\Pi_B(0) - \Pi_B(2r^*) = 2 \left[\Pi_B(r_1^B) - \Pi_B(r_1^B + r^*) \right]$ implies $r_1^B = \frac{r^*}{2}$. Thus, for the deviation not to be profitable we only require

$$r^* D(p^M(2r^*)) \geq \frac{r^*}{2} D\left(p^M\left(\frac{r^*}{2} \right) \right).$$

When L_B is 0, $r^* = 0$ and the result holds trivially. The derivative of the profit functions evaluated at $r^* = 0$ are $D(p^M(0))$ and $\frac{1}{2} D(p^M(0))$ for the left-hand side and the right-hand

side expression, respectively. Thus, the deviation is not profitable when L_B is sufficiently small.

We now show that there is no other symmetric pure-strategy equilibrium when the litigation constraint is relevant. First, notice that if $r_1 = r_2$ are lower than r^*, each firm has incentives to increase its royalty since their problem is the same as they would face if they were unconstrained and royalties are strategic substitutes. If, instead, $r_1 = r_2 = \tilde{r}$ are higher than r^* each firm obtains profits

$$\frac{1}{2}\left[g(x)\tilde{r}D(p^M(2\tilde{r})) - L_U\right] + \frac{1}{2}\left[g(x)\tilde{r}D(p^M(2\tilde{r})) + (1 - g(x))\left[g(x)\tilde{r}D(p^M(2\tilde{r})) - L_U\right]\right]$$

where each firm is brought to court first with probability $\frac{1}{2}$ and the second firm is sued only if the downstream producer succeeds against the first. Notice that in this case it is always optimal for one firm, say patentee 1, to undercut the other patentee. As a result profits increase to

$$g(x)\tilde{r}D(p^M(2\tilde{r})) + (1 - g(x))\left[g(x)\tilde{r}D(p^M(2\tilde{r})) - L_U\right].$$

$\square$

Proof of Lemma 7: Define

$$\Phi(r_i, x_i, L_B, R_{-i}) \equiv (1 - g(x_i))\left[\Pi_B(R_{-i}) - \Pi_B(R_{-i} + r_i)\right] +$$
$$g(x_i)h(x_i)\left[\Pi_B(R_{-i} + \rho(x_i, r_i, R_{-i})) - \Pi_B(R_{-i} + r_i)\right] - L_B$$

Obviously, $\frac{\partial \Phi}{\partial L_B} = -1$. We can also compute

$$\frac{\partial \Phi}{\partial r_i} = -(1 - g(x_i))\Pi'_B(R_{-i} + r_i) + g(x_i)h(x_i)\left[\Pi'_B(R_{-i} + \rho(x_i, r_i, R_{-i}))\frac{\partial \rho}{\partial r_i} - \Pi'_B(R_{-i} + r_i)\right]$$

$$\frac{\partial \Phi}{\partial R_{-i}} = (1 - g(x_i))\left[\Pi'_B(R_{-i}) - \Pi'_B(R_{-i} + r_i)\right]$$
$$+ g(x_i)h(x_i)\left[\Pi'_B(R_{-i} + \rho(x_i, r_i, R_{-i}))\left(1 + \frac{\partial \rho}{\partial R_i}\right) - \Pi'_B(R_{-i} + r_i)\right]$$

Given that Π_B is convex, $\rho(x_i, r_i, R_{-i}) \leq r_i$ and the assumption that $\rho(x_i, r_i, R_{-i})$ is independent of r_i and R_{-i}) we can show that $\frac{\partial \Phi}{\partial r_i} \geq 0$ and $\frac{\partial \Phi}{\partial R_{-i}} \leq 0$. $\square$

Proof of Lemma 8: Consider the case of a single innovator. Given a total royalty rate R the firm set a fixed fee F that claws back all the surplus of the firm. That is $(1 + \mu)F = \Pi_B(R)$. This means that the firm would choose a royalty rate to maximize

$$\max_R R\tilde{D}(R) + \frac{\Pi_B(R)}{1 + \mu}.$$

The first order condition results in an optimal royalty rate

$$R^m \tilde{D}'(R^m) + \tilde{D}(R^m)\frac{\mu}{1+\mu} = 0.$$

In the case of two innovators, firm i obtains a fixed fee $F_i = \frac{\Pi_B(r_1+r_2)}{1+\mu} - F_j$ for $j \neq i$. As a result, the maximization of firm i is

$$\max_{r_i} \; r_i \tilde{D}(r_1 + r_2) + \frac{\Pi_B(r_1 + r_2)}{1+\mu} - F_j,$$

with a first order condition that in a symmetric equilibrium can be characterized as

$$r^u \tilde{D}'(2r^u) + \tilde{D}(2r^u)\frac{\mu}{1+\mu} = 0.$$

It is immediate that the log-concavity of $\tilde{D}(R)$ implies that $R^M < 2r^u$. Otherwise,

$$R^M = -\frac{\tilde{D}(R^M)}{\tilde{D}'(R^M)}\frac{\mu}{1+\mu} \leq -\frac{\tilde{D}(2r^u)}{\tilde{D}'(2r^u)}\frac{\mu}{1+\mu} = r^u,$$

which is a contradiction unless $R^M = r^u = 0$ which cannot occur if $\mu > 0$. Finally, notice that the first order condition in both cases is increasing in μ implying that the higher the distortion from using fixed fees the higher the royalty rate. $\qquad \square$

Proof of Proposition 9: Using the same arguments as in the benchmark model, a pure-strategy equilibrium with royalty rates r_1^* and r_2^* can exist only if

$$(1 - g(x_2)) \left[\Pi_B(r_1^*) - \Pi_B(r_1^* + r_2^*) + (1+\mu)F_2\right] < L_B.$$

Otherwise, innovator 1 has incentives to lower the royalty rate and induce litigation on the patent of innovator 2. Furthermore, the optimal choice of innovator 2 has to be interior in the sense that it must be the solution to

$$\max_{r_2} r_2 \tilde{D}(r_1 + r_2) + F_2$$
$$\text{s.t. } \Pi_B(r_1 + r_2) - (1+\mu)(F_1 + F_2) \geq 0,$$
$$(1 - g(x_2)) \left[\Pi_B(r_1) - \Pi_B(r_1 + r_2) + (1+\mu)F_2\right] < L_B.$$

It is easy to see that this problem can be rewritten as

$$\max_{r_2} r_2 \tilde{D}(r_1 + r_2) + \frac{\Pi_B(r_1 + r_2)}{1+\mu} - A,$$

where $A = \max\{\Pi_B(r_1) - \frac{L_B}{1-g(x_2)}, (1+\mu)F_1\}$. This means that the first order condition is identical to the one that arises in the case of ironclad patents,

$$r_2 \tilde{D}'(r_1 + r_2) + \tilde{D}(r_1 + r_2)\frac{\mu}{1+\mu} = 0,$$

leading to a symmetric equilibrium (in royalty rates) $r_1^* = r_2^* = r^u$. $\qquad \square$