Royalty Stacking and Validity Challenges: The Inverse Cournot Effect

Online Appendix

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This document contains two sections that expands on some of the results in the main body of the paper. The first section discusses a parametric example of the basic model setup. The second section provides an extensive analysis to the case where litigation might occur in equilibrium.

B A Parametric Example

Consider the case where the downstream demand corresponds to a unique consumer with a valuation for one unit of the good. With probability $\alpha \in (0,1)$ this valuation is 1. With probability $1-\alpha$ the valuation is $v < 1$. Furthermore, we assume that the downstream firm chooses the price after the valuation has been realized. This timing implies that the downstream producer will always choose a price equal to the realized valuation of the consumer. That is, given R the downstream producer captures all the surplus without generating the losses associated to double marginalization. As a result, expected downstream profits $\Pi_B(R)$ can be computed as

$$\Pi_B(R) = \begin{cases} \alpha + (1-\alpha)v - R & \text{if } R \leq v, \\ \alpha(1-R) & \text{if } R \in (v,1], \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

These profits are decreasing and weakly convex in R.[1] Notice that the demand is weakly log-concave in the price as expected from Assumption 1. However, the fact that profits

[1] A dead-weight loss would arise if we assumed that the downstream producer chose the price before the demand is realized. In that case, the threshold value on R in the profit function $\Pi_B(R)$ would change. That is, $p^M(R) = v$ if and only if $R \leq \tilde{R} \equiv \frac{v-\alpha}{1-\alpha} < v$. Since double marginalization does not interact with the mechanisms explored in this paper (see Assumption 1), the main results would go through under this alternative assumption although at the cost of an increasing technical complexity.

are not linear everywhere is enough for our results to go through.

We start by characterizing the royalty rate that maximizes joint profits for the upstream patent holders when their portfolio is sufficiently strong so that $g(x_1) = g(x_2) = 1$. This royalty rate will be used as a benchmark for the case in which innovators decide independently.

Proposition 1. *Under the two-point demand function, when $g(x_1) = g(x_2) = 1$ there is a continuum of undominated pure-strategy equilibria. The corresponding royalty rates (r_1^u, r_2^u) can be characterized as follows:*

1. *If $v \geq \frac{2\alpha}{1+\alpha}$, $R^u = r_1^u + r_2^u = v$ with $r_i^u \leq \frac{v-\alpha}{1-\alpha}$ for $i = 1, 2$,*
2. *If $v \leq \frac{1+\alpha}{2}$, $R^u = r_1^u + r_2^u = 1$ with $r_i^u \geq \frac{v-\alpha}{1-\alpha}$ for $i = 1, 2$.*

Both kinds of equilibria co-exist when $\frac{2\alpha}{1+\alpha} \leq v \leq \frac{1+\alpha}{2}$. All equilibria imply royalty stacking when $\alpha \leq v < \frac{2\alpha}{1+\alpha}$.

Proof. Regarding the first case, contingent on selling with probability 1 the sum of royalties must be equal to v or otherwise any patent holder would deviate and increase the royalty rate. Hence, take r_1^u and $r_2^u = v - r_1^u$ and suppose without loss of generality that $r_1^u \geq \frac{v}{2} \geq r_2^u$. The optimal deviation for patentee i is $\hat{r}_i = 1 - r_j^u$ for $j \neq i$ and it would be unprofitable if $v - r_j^u \geq \alpha(1 - r_j^u)$ or $r_j^u \leq \frac{v-\alpha}{1-\alpha}$. Such a combination of royalties is only possible as long as $\frac{v}{2} \leq r_1^u \leq \frac{v-\alpha}{1-\alpha}$ or $v \geq \frac{2\alpha}{1+\alpha}$.

For the second case, take r_1^u and $r_2^u = 1 - r_1^u$ and suppose without loss of generality that $r_1^u \geq \frac{1}{2} \geq r_2^u$. The optimal deviation for patentee i is $\hat{r}_i = v - r_j^u$ for $j \neq i$ if it leads to a positive royalty and it would be unprofitable if $\alpha(1 - r_j^u) \geq v - r_j^u$ or $r_j^u \geq \frac{v-\alpha}{1-\alpha}$. Such a combination of royalties will be possible as long as $\frac{v-\alpha}{1-\alpha} \leq r_2^u \leq \frac{1}{2}$ or $v \leq \frac{1+\alpha}{2}$.

Finally, notice that $\frac{2\alpha}{1+\alpha} < \frac{1+\alpha}{2}$ for all $\alpha \in [0, 1]$ so both equilibria can co-exist. □

Intuitively, the equilibrium with a total royalty of 1 is likely to exist when v is small and α is sufficiently close to 1. A deviation might exist if any patent holder prefers to

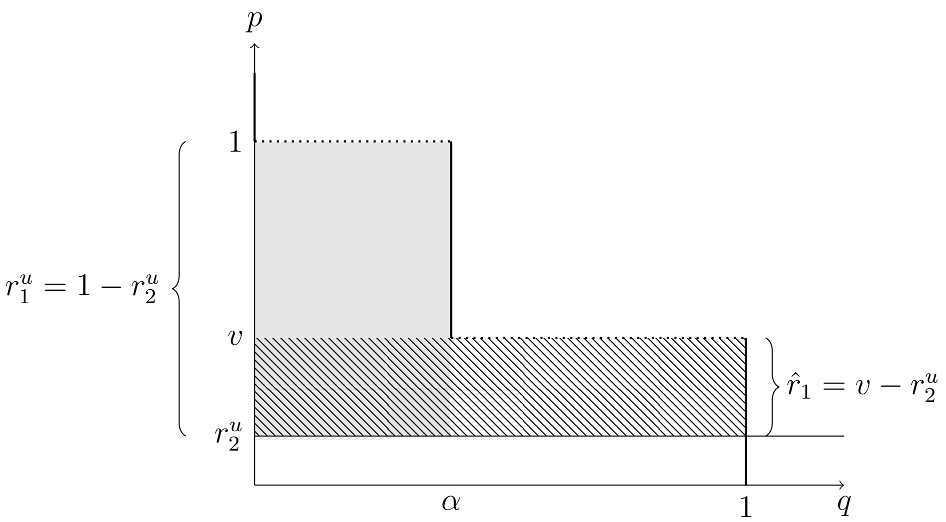

Figure 1: Equilibrium with $r_1^u + r_2^u = 1$. Profits for patent holder 1 correspond to the gray area. The striped area indicates the profits under the optimal deviation.

decrease the royalty rate in order to cater the consumer regardless of her valuation. This deviation is illustrated in Figure 1. Given r_2^u, innovator 1 can choose $r_1^u = 1 - r_2^u$ or deviate and choose $\hat{r}_1 = v - r_2^u$ so that the probability of selling increases from α to 1. Such a deviation is unprofitable if r_2^u is sufficiently large and, thus, the low $\hat{r}_1$ does not allow the firm to benefit from the increase in sales. In the limit, when $v = 0$ or $\alpha = 1$ this equilibrium holds for any combination of royalties that sums up to 1.

Similarly, equilibria with a total royalty equal to $R^u = v$ are likely to exist when v is sufficiently high and α is sufficiently small. This time a deviation aims to capture the additional surplus when consumer valuation is 1, even if this surplus is materialized only with probability α. To prevent this deviation each innovator must set a modest royalty so that the other firm already obtains sufficiently high profits in equilibrium, thus reducing the appeal of raising the royalty rate and reducing the probability of sale. In the limit, when $v = 1$ or $\alpha = 0$ any combination of royalty rates that sums up to v would constitute an equilibrium. Such coordination would also maximize social welfare.

In contrast, a total royalty $R = v$ would be chosen by a single innovator owning both patents if and only if $v > \alpha$. Royalty stacking, which here takes the form of a total royalty rate equal to 1 when joint profit maximization requires $R^M = v$, arises as a Nash equilibrium when $\alpha \leq v < \frac{1+\alpha}{2}$ and it is unique when $\alpha \leq v < \frac{2\alpha}{1+\alpha}$.

B.1 One Constrained Patent Holder

Suppose now that $g(x_1) = 1$ and $g(x_2) < 1$ so that the downstream producer may only be interested in litigating the portfolio of innovator 2. We restrict our discussion to the case where $v > \alpha$ so that, according to Proposition 1, in the previous benchmark a combination of royalties for which $r_1^* + r_2^* = 1$ constituted an equilibrium with royalty stacking. Once the threat of litigation is accounted for, such an equilibrium may fail to exist for two reasons. First, given $r_1^* + r_2^* = 1$, the downstream firm will obtain higher profits by going to court if

$$r_2^* > \bar{r}_2 = \begin{cases} \frac{L_B}{\alpha(1-g(x_2))} & \text{if } \frac{L_B}{1-g(x_2)} < \alpha(1-v), \\ (1-\alpha)(1-v) + \frac{L_B}{1-g(x_2)} & \text{otherwise.} \end{cases} \tag{2}$$

Second, if $r_2^* \leq \bar{r}_2$, innovator 1 might benefit from deviating to a royalty below r_1^* which induces litigation. Using a similar logic as in Lemma 3, we can show that when the litigation cost L_B is sufficiently low there is a threshold $\bar{r}_1 > 0$ such that litigation will occur if $r_1 < \bar{r}_1$. The next lemma characterizes the region under which litigation may occur as a function of r_1.

Lemma 2. *Under the two-point demand function, if $r_2 > \bar{r}_2$ there is no equilibrium with royalty stacking and no litigation. If*

$$r_2 \leq \underline{r}_2 = \begin{cases} \frac{L_B}{1-g(x_2)} & \textit{if } \frac{L_B}{1-g(x_2)} \leq v, \\ \frac{L_B}{\alpha(1-g(x_2))} - \frac{1-\alpha}{\alpha}v & \textit{otherwise,} \end{cases} \tag{3}$$

innovator 2 will not be brought to court for any $r_1 \geq 0$. If $r_2 \in (\underline{r}_2, \bar{r}_2]$, litigation will occur if

$$r_1 < \bar{r}_1(r_2) = v + \frac{\alpha}{1-\alpha}r_2 - \frac{L_B}{(1-\alpha)(1-g(x_2))} \leq v. \tag{4}$$

Proof. From the argument in the text it is immediate that for $r_2 > \bar{r}_2$ an equilibrium without litigation and with royalty stacking cannot arise, since for all $r_1 = 1 - r_2$ litigation will be profitable for the downstream firm. For the rest of the arguments, it is useful to distinguish two cases depending on the relationship between v and $\frac{L_B}{1-g(x_2)}$.

Suppose that $\frac{L_B}{1-g(x_2)} \leq v$. First notice that litigation will not occur for any value of r_1 if and only if $r_2 \leq \underline{r}_2 = \frac{L_B}{1-g(x_2)}$. From Lemma 3, the incentives to litigate are highest when $r_1 = 0$ and, in that case, the expected profits from going to court are $(1-g(x_2))r_2 \leq L_B$. Consider now the case $r_2 \in \left(\frac{L_B}{1-g(x_2)}, \bar{r}_2\right)$. By definition, when $r_2 < \bar{r}_2$ a royalty $r_1 = 1 - r_2$ induces litigation. Even if r_2 is sufficiently close to v a royalty $r_1 = 0$ litigation will always be profitable for the downstream producer since $(1-g(x_2))\left[\Pi_B(0) - \Pi_B(v)\right] = (1-g(x_2))v > L_B$. The value of r_2 for which the downstream firm is indifferent between litigating or not is defined by (4).

Consider now the case in which $v < \frac{L_B}{1-g(x_2)}$. First suppose that $r_2 \leq \underline{r}_2 = \frac{L_B}{\alpha(1-g(x_2))} - \frac{1-\alpha}{\alpha}v < \frac{L_B}{1-g(x_2)}$. In that case, even r_1 will not induce litigation since the downstream profits from going to court will be $(1-g(x_2))\left[(1-\alpha)v + \alpha r_2\right] \leq L_B$. Suppose now that $r_2 \in \left(\frac{L_B}{\alpha(1-g(x_2))} - \frac{1-\alpha}{\alpha}v, \bar{r}_2\right)$. By definition, when $r_2 < \bar{r}_2$ a royalty of $r_1 = 1 - r_2$ induces litigation. If, instead, r_2 is sufficiently close to $\frac{L_B}{\alpha(1-g(x_2))} - \frac{1-\alpha}{\alpha}v < v$ a royalty $r_1 = 0$ will induce litigation since the downstream profits of going to court are $(1-g(x_2))\left[(1-\alpha)v + \alpha r_2\right] > L_B$. The value of r_2 for which the downstream firm is indifferent between litigating or not is defined by (4). □

Innovator 1 might benefit from lowering the royalty rate below $\bar{r}_1$ if, by causing litigation against patentee 2, the quantity sold expands from α to 1, which would occur with probability $1 - g(x_2)$. Hence, a profitable deviation $\hat{r}_1$ must be lower than v. Since $\bar{r}_1(\bar{r}_2) \leq v$ it follows that the optimal deviation for innovator 1 when patentee 2 sets $r_2^* \leq \bar{r}_2$ is the highest royalty rate which guarantees that the patent of innovator 2 is litigated, $\hat{r}_1 = \bar{r}_1(r_2^*)$.[2] Innovator 1's profits in that case would become

$$\hat{\Pi}_1 = \left[\alpha + (1-\alpha)(1-g(x_2))\right]\hat{r}_1. \tag{5}$$

That is, a deviation will lead to profits equal to $\hat{r}_1$ either because the valuation of the consumer is 1 or because the valuation is v but the patent of innovator 2 has been

[2] More precisely, given our assumptions, $\hat{r}_1$ should be slightly lower than $\bar{r}_1(r_2^*)$.

invalidated in court. This deviation will take place if profits, $\hat{\Pi}_1$, are higher than those in the candidate equilibrium, $\Pi_1^* = \alpha r_1^*$. Notice that the lower are r_1^* or $g(x_2)$ the more binding this condition becomes. The next proposition characterizes the circumstances under which $\Pi_1^* \geq \hat{\Pi}_1$ cannot hold while, as required by Proposition 1, $r_2^* \geq \frac{v-\alpha}{1-\alpha}$. In those situations, an equilibrium with royalty stacking and no litigation will fail to exist.

Proposition 3. *Consider the two-point demand function case and suppose $v > \alpha$. If $\frac{L_B}{1-g(x_2)} < \frac{v-\alpha}{1-\alpha}$ and $g(x_2)$ is sufficiently small, there is no equilibrium with royalty stacking and no litigation. If L_U is sufficiently large, only the efficient equilibrium exists, which involves $r_2^* \leq \frac{L_B}{1-g(x_2)} < v$ and $r_1^* = v - r_2^*$.*

Proof. From Proposition 1, a necessary condition for a royalty-stacking equilibrium with not litigation to exist is that $r_1^* + r_2^* = 1$ and $r_2^* \geq \frac{v-\alpha}{1-\alpha}$ or, else, patent holder 1 would have incentives to lower its royalty rate.

Furthermore, a second deviation consisting in choosing a royalty slightly lower than $\bar{r}_1(r_2^*)$ might be profitable for patent holder 1 if it leads to profits

$$\hat{\Pi}_1 = [\alpha + (1-\alpha)(1-g(x_2))]\, \bar{r}_1(r_2^*) > \alpha r_1^*.$$

This condition holds if

$$r_2^* < \rho(G) \equiv \frac{(1-\alpha)(\alpha - Gv) + G\frac{L_B}{1-g(x_2)}}{\alpha(G + (1-\alpha))},$$

where $G \equiv \alpha + (1-\alpha)(1-g(x_2)) \in [\alpha, 1]$. Thus, in instances in which $\rho(G) < \frac{v-\alpha}{1-\alpha}$ an equilibrium with royalty stacking will fail to exist. This inequality implies that

$$G < G^*\left(\frac{L_B}{1-g(x_2)}\right) = \frac{\alpha(1-\alpha)(1-v)}{(1-\alpha)\left(v - \frac{L_B}{1-g(x_2)}\right) - \alpha^2(1-v)}.$$

This function is increasing in $\frac{L_B}{1-g(x_2)}$ and $G^*\left(\frac{L_B}{1-g(x_2)}\right) < G^*\left(\frac{v-\alpha}{1-\alpha}\right) = 1$. Hence, there is always $g(x_2)$ sufficiently small so that the deviation will be optimal.

We now consider conditions under which an equilibrium with $R = v$ exists. Consider the case $r_2^* = \frac{L_B}{1-g(x_2)} < \frac{v-\alpha}{1-\alpha}$ and $r_1^* = v - r_2^*$. From (3), r_2^* avoids litigation and by

Proposition 1 patentee 1 has no incentive to deviate. Thus, the only deviation we need to consider from patentee 2 is such that $R > v$. However, notice that

$$r_1^* = v - \frac{L_B}{1-g(x_2)} = v + \frac{\alpha}{1-\alpha} r_2^* - \frac{L_B}{(1-\alpha)(1-g(x_2))} = \bar{r}_1(r_2^*),$$

and so any higher r_2 will induce litigation. Hence, an equilibrium in pure strategies exists if and only if such a deviation is not profitable

$$\frac{L_B}{1-g(x_2)} \geq \alpha g(x_2) \left(1 - v + \frac{L_B}{1-g(x_2)}\right) - L_U.$$

This condition is guaranteed if $g(x_2)$ is sufficiently small or L_U sufficiently large. □

This result indicates that when L_B and/or $g(x_2)$ are sufficiently low, royalty stacking will not arise in an equilibrium without litigation. In order to interpret this result it is useful to start by considering the case under which such an equilibrium with royalty stacking may exist. From (3) we know that if $r_2^* \leq \frac{L_B}{1-g(x_2)}$ the Inverse Cournot effect has no bite since there is no positive value of $\hat{r}_1$ that triggers litigation. When $\frac{L_B}{1-g(x_2)} \geq \frac{v-\alpha}{1-\alpha}$ it is also possible to find $r_2^* \geq \frac{v-\alpha}{1-\alpha}$, satisfying the conditions of Proposition 1. Hence, it is optimal for innovator 1 to choose $r_1^* = 1 - r_2^*$ and an equilibrium with royalty stacking will arise in that case.

When $\frac{L_B}{1-g(x_2)} < \frac{v-\alpha}{1-\alpha}$, however, a deviation from the royalty-stacking equilibrium may exist. Starting from a combination of royalties (r_1^*, r_2^*) with $r_i^* \geq \frac{v-\alpha}{1-\alpha}$ for $i = 1, 2$, patent holder 1 trades off a decrease in the royalty to $\bar{r}_1(r_2^*)$ with an increase in the probability of sale from α to $\alpha + (1-\alpha)(1-g(x_2))$. The previous proposition shows that if $g(x_2)$ is sufficiently small this expansion effect dominates and the deviation is profitable. The reason for this result is, precisely, that when $v > \alpha$ eliminating royalty stacking raises total profits and the smaller is $g(x_2)$ the larger is the proportion of that increase that innovator 1 can appropriate.

The second part of the proposition also indicates that when the probability of success in court of innovator 2 is small two results concur. First, the royalty rate is commensurate with the strength of the patent portfolio and the cost of challenging those rights by the downstream producer, $r_2^* \leq \frac{L_B}{1-g(x_2)}$. This result arises from the fact that when $g(x_2)$ is small innovator 2 must choose a low royalty rate to discourage the downstream producer from engaging in litigation that will, most likely, result in a zero royalty. Second, and more interestingly, the joint profit maximizing equilibrium, consisting of $R^M = v$, may exist. The reason is that the low value of r_2 makes innovator 1 the residual claimant of the surplus generated. This can be seen using Figure 1, where the lower is r_2 the more innovator 1 internalizes the losses that a deviation towards a larger royalty rate may entail.

B.2 Two Constrained Patent Holders

Suppose now that both firms have an identical patent holdings that does not confer full protection against litigation, $g(x_1) = g(x_2) = g(x) < 1$. As in the previous case we focus on the situation in which royalty stacking was an equilibrium when no litigation was feasible, $v > \alpha$. We study whether litigation affects the existence of an equilibrium with royalty stacking, so that $r_1^* + r_2^* = 1$. As in the general case, it is enough to focus on the symmetric case in which $r_1^* = r_2^* = 1/2$ as if this equilibrium did not exist no asymmetric equilibrium would exist either.[3] We also explained that in the symmetric case it will never be optimal for the downstream producer to bring to court only one of the innovators. The next lemma characterizes the threshold values of $\hat{r}_1$ for which innovator 1 expects to be sued in case the other patentee loses in court.

Lemma 4. *Under the two-point demand function with $v > \alpha$, suppose that for $r_1^* = r_2^* = 1/2$ it is not profitable for the downstream producer to engage in litigation. If by deviating to $\hat{r}_1 < r_1^*$ innovator* 2 *is sued and its patent invalidated, innovator* 1 *will also be sued if and only if $\hat{r}_1 > \frac{L_B}{1-g(x)}$.*

[3] As discussed in previous sections, an equilibrium may fail to exist because one of the royalty rates is too low and, as a result, either the innovator decides to deviate and raise it even at the cost of being sued or the other patentee may benefit from lowering its own royalty rate and serve the whole market. By focusing on the symmetric royalty rate we are minimizing the profitability of these deviations.

Proof. First notice that if patent holder 2 loses in court patent holder 1 will be brought to court if and only if

$$\Pi_B(0) - \Pi_B(\hat{r}_1) > \frac{L_B}{1-g(x)}$$

or $\hat{r}_1 > \frac{L_B}{1-g(x)}$. Also notice that, from the arguments in the text, if originally it was not optimal to engage in litigation it has to be that

$$\Pi_B(1/2) - \Pi_B(1) \leq \frac{L_B}{1-g(x)}.$$

Patent holder 1 would be sued after downstream producer loses against patent holder 2 if

$$\Pi_B(1/2) - \Pi_B(1/2 + \hat{r}_1) > \frac{L_B}{1-g(x)}$$

which is incompatible with the previous condition. □

The deviations that this lemma characterizes determine two regions depending on whether $\hat{r}_1$ is higher or lower than $\frac{L_B}{1-g(x)}$. Both deviations are less profitable than in the case in which $g(x_1) = 1$, albeit for slightly different reasons. In one of the regions, by choosing a low $\hat{r}_1$, innovator 1 eludes litigation but at the cost of a significant reduction in licensing revenues. In the second region, when $\hat{r}_1$ is higher, the lower profitability of the deviation arises from the probability that the innovator might not receive any licensing revenues from its patent if it is declared invalid, together with the corresponding litigation costs. In particular, in this last region, the profits from a deviation are

$$\hat{\Pi}_1 = g(x)\alpha\hat{r}_1 + (1-g(x))\left[g(x)\hat{r}_1 - L_U\right].$$

That is, when the patent of the other innovator is upheld in court the expected quantity is α. If, instead, the portfolio of innovator 2 is invalidated and the downstream producer also decides to sue innovator 1, the quantity sold is 1 but the royalty $\hat{r}_1$ is only paid if the corresponding patent is upheld in the second trial.

We now illustrate how the risk of a litigation cascade might foster the existence of an equilibrium with royalty stacking and no litigation. Take the case in which L_U is very large so that the threat of litigation is particularly relevant for the upstream patent holders, and consider the situation in which $v \leq \frac{1}{2}$. Given $r_1^* = r_2^* = 1/2$, two conditions must be satisfied for such an equilibrium to exist. First, using equation (4), the downstream producer should not be interested in going to court, which in this case it implies

$$\frac{L_B}{1-g(x_2)} \geq \frac{1}{2-g(x)} \left[g(x)\frac{\alpha}{2} + (1-g(x)(\alpha+(1-\alpha)v) \right]. \tag{6}$$

Second, the cost of a litigation cascade implies that the optimal deviation of innovator i, for $i = 1, 2$, involves $\hat{r}_i = \min \left\{ v, \frac{L_B}{1-g(x)} \right\}$ and such a deviation is unprofitable if and only if $\hat{\Pi}_1 \leq \Pi^*$ or

$$[\alpha + (1-\alpha)(1-g(x))]\, \hat{r}_i \leq \frac{\alpha}{2}. \tag{7}$$

Notice that because, as in the case of one constrained patent holder, $\hat{r}_i \leq v$ the expected demand expands if the patent of the other innovator is invalidated.

These two conditions provide a lower and upper bound, respectively, on $\frac{L_B}{1-g(x)}$ for an equilibrium with royalty stacking and no litigation to exist. That is, the litigation costs of the downstream producer must be sufficiently large to discourage this firm from litigating but they must also be sufficiently small so that the decrease in the royalty rate necessary for a deviating firm to fend off litigation is large.

Although the previous conditions are highly non-linear in the main parameters of the model it is easy to find combinations that satisfy them. More interestingly, we can also find situations in which this equilibrium with a total royalty equal $R^* = 1$ is sustainable when both innovators have a very strong or a very weak patent but not in the case in which the patents have an asymmetric strength.

Example 1. *Consider the parameter values $\alpha = 0.1$, $v = 0.3$, $g(x_2) = 0.68$, $L_B = 0.035$, and L_U sufficiently large. If litigation were not possible, the parameter values would satisfy the conditions of Proposition 1 and an equilibrium with royalty stacking, $R^u = 1$, would exist.*

Next, consider the case in which $g(x_1) = 1$ *so that only the second patent holder is potentially constrained. By construction,* $\frac{L_B}{1-g(x_2)} < \frac{v-\alpha}{1-\alpha}$*, and it can be verified that innovator* 1 *has incentives to deviate from any candidate equilibrium* (r_1^*, r_2^*) *and choose* $\bar{r}_1(r_2^*)$*, so that the royalty-stacking will not emerge in this case.*

Finally, consider the case in which $g(x_1) = g(x_2) = 0.68$*. Equations* (6) *and* (7) *are satisfied and, thus, the royalty-stacking equilibrium exists when both innovators are similarly constrained.*

The previous example illustrates that, as in the main sections of the paper, once we introduce litigation in the model the royalty rate is not necessarily monotonic in the strength of the patents. When patents are weaker but more evenly distributed the royalty-stacking problem might actually become more relevant.

B.3 Ad-Valorem Royalties

In this section we show that, under ad-valorem royalties, royalty stacking can also be eliminated. We do so in the context of the parametric example of this section where we assume that the downstream producer faces a marginal cost $c \in (0, v)$. Under ad-valorem royalties downstream profits can be written as

$$\Pi_B(S) = \begin{cases} (1-S)(\alpha + (1-\alpha)v) - c & \text{if } S \leq 1 - \frac{c}{v}, \\ \alpha(1-S-c) & \text{if } S \in \left(1-\frac{c}{v}, 1-c\right], \\ 0 & \text{otherwise,} \end{cases} \tag{8}$$

We can now write a counterpart of Proposition 1.

Proposition 5. *Under the two-point demand function, there exists a threshold* $\tilde{v}$ *such that joint profit maximization implies a royalty* $S^M = 1-c$ *if* $v > \tilde{v}$ *and* $S^M = 1-\frac{c}{v}$ *otherwise. Under competition there is a continuum of undominated pure-strategy equilibria. There exist values* $\underline{v}$ *and* $\overline{v}$*, such that* $\tilde{v} \leq \underline{v} < \overline{v}$ *so that the equilibrium ad-valorem royalty rates* (s_1^u, s_2^u) *can be characterized as follows:*

1. *If* $v \geq \underline{v}$*,* $S^u = s_1^u + s_2^u = 1 - \frac{c}{v}$ *with* $s_i^u \leq 1 - \frac{(1-2\alpha)cv+\alpha c}{(1-\alpha)\,v^2}$ *for* $i = 1, 2$*.*

2. If $v \leq \overline{v}$, $S^u = s_1^u + s_2^u = 1 - c$ with $s_i^u \geq 1 - \frac{(1-2\alpha)cv+\alpha c}{(1-\alpha)v^2}$ for $i = 1, 2$.

As a result, royalty stacking emerges in equilibrium when $\tilde{v} < v < \underline{v}$.

Proof. As in the case of per-unit royalties only two ad-valorem rates can maximize joint profits, $1 - \frac{c}{v}$ and $1 - c$. The low rate dominates if

$$\left(1 - \frac{c}{v}\right)(\alpha + (1-\alpha)v) \geq (1-c)\alpha$$

or

$$v \leq \tilde{v} \equiv \frac{(1-2\alpha)\,c + \sqrt{(4\alpha^2 - 4\alpha + 1)\,c^2 + (4\alpha - 4\alpha^2)\,c}}{2(1-\alpha)}.$$

Regarding the Nash equilibria, suppose that patent holder $j = 1, 2$ chooses s_j. Patent holder i will prefer $s_i = 1 - \frac{c}{v} - s_j$ to $s_i = 1 - c - s_j$ if

$$\left(1 - \frac{c}{v} - s_j\right)(\alpha + (1-\alpha)v) \geq (1 - c - s_j)\alpha$$

or $s_j \leq \bar{s} \equiv \frac{(1-\alpha)\,v^2 - (1-2\,\alpha)\,c\,v - \alpha\,c}{(1-\alpha)\,v^2}$. Hence, for this equilibrium to exist we require that $2\bar{s} \geq 1 - \frac{c}{v}$ or

$$v \geq \underline{v} \equiv \frac{(1-3\alpha)\,c + \sqrt{(9\alpha^2 - 6\,\alpha + 1)\,c^2 + (8\alpha - 8\alpha^2)\,c}}{2(1-\alpha)}.$$

Similarly, an equilibrium with $S^u = 1 - c$ would exist if $s_j \geq \bar{s}$ and $2\bar{s} \leq 1 - c$ which can occur if

$$v \leq \overline{v} \equiv \frac{(1-2\,\alpha)\;c + \sqrt{(2\,\alpha^2 - 2\,\alpha + 1)\;c^2 + (2\,\alpha - 2\,\alpha^2)\,c}}{(1-\alpha)\,(1+c)}.$$

Comparison of the threshold expressions lead to $\tilde{v} \leq \underline{v} < \overline{v}$ if $\alpha < 0$ and $c \in (0, v)$. □

As in the previous case, royalty stacking arises when v takes an intermediate value. Innovators individually charge a total royalty rate higher than what joint maximization would find optimal. The proof provides the specific expressions for the different thresholds.

We turn now to the case where innovator 1 has a portfolio of strength x_1 such that $g(x_1) = 1$ whereas x_2 is such that $g(x_2) < 1$. First notice that a necessary condition for an equilibrium with royalty stacking to exist, $s_1^* + s_2^* = 1 - c$, is that the downstream producer does not have incentives to sue innovator 2. In particular, this implies that

$$(1 - g(x_2)) \left[\Pi_B(1 - c - s_2^*) - \Pi_B(1 - c)\right] \leq L_B.$$

Two cases arise depending on whether the royalty of innovator 1, $s_1^* = 1 - c - s_2^*$, is greater than $1 - \frac{c}{v}$ or not. As a result, litigation by the downstream producer will not be profitable if

$$s_2^* \leq \bar{s}_2 = \begin{cases} \frac{L_B}{\alpha(1-g(x_2))} & \text{if } \frac{L_B}{1-g(x_2)} < \alpha\left(\frac{c}{v} - c\right), \\ c\frac{(1-\alpha)(1-v)}{\alpha+(1-\alpha)v} + \frac{L_B}{(\alpha+(1-\alpha)v)(1-g(x_2))} & \text{otherwise.} \end{cases} \tag{9}$$

For a given royalty rate s_2 set by innovator 2 we can define, using equation (7), the threshold royalty rate $\bar{s}_1$ as

$$\bar{s}_1(s_2) = 1 - \frac{c}{v} - \frac{L_B}{(1 - g(x_2))(1 - \alpha)v} + \frac{\alpha}{(1 - \alpha)v} s_2 \text{ if } \underline{s}_2 \leq s_2 \leq \bar{s}_2.$$

where for any $s_2 < \bar{s}_2$ we have $\bar{s}_1 \leq 1 - \frac{c}{v}$. As in the previous case, if $s_1 < \bar{s}_1(s_2)$ the patent of innovator 2 will be litigated by the downstream producer. The lower threshold is defined as the highest value of s_2 for which it is not worthwhile to sue innovator 2 even when innovator 1 chooses $s_1 = 0$ and it can be written as

$$s_2 < \underline{s}_2 = \begin{cases} \frac{L_B}{(1-g(x_2))(\alpha+(1-\alpha)v)} & \text{if } \frac{L_B}{1-g(x_2)} \leq (\alpha + (1-\alpha)v)\left(1 - \frac{c}{v}\right), \\ \frac{L_B}{\alpha(1-g(x_2))} - \frac{1-\alpha}{\alpha}(v - c) & \text{otherwise.} \end{cases} \tag{10}$$

Given s_2, innovator 1 will have incentives to deviate if, by choosing $s_1 \leq \bar{s}_1(s_2)$, profits increase due to the increase in quantity when the patent of innovator 2 is invalidated. The next proposition shows that, as in the case in which royalties were paid per-unit, the royalty stacking equilibrium fails to exist when the portfolio of innovator 2 is sufficiently weak.

Proposition 6. *Suppose that $v > \tilde{v}$. If $\frac{L_B}{(1-g(x_2))(\alpha+(1-\alpha)v)} < 1 - \frac{(1-2\alpha)cv+\alpha c}{(1-\alpha)v^2}$ and $g(x_2)$ is sufficiently small, there is no pure strategy equilibrium with royalty stacking.*

Proof. From Proposition 5, a necessary condition for a royalty-stacking equilibrium to exist is that $s_1^* + s_2^* = 1 - c$ and $s_2^* \geq 1 - \frac{(1-2\alpha)cv+\alpha c}{(1-\alpha)v^2}$ or, else, patent holder 1 would have incentives to lower its royalty rate.

Furthermore, a second deviation consisting in choosing a royalty slightly lower than $\bar{s}_1(s_2^*)$ might be profitable for patent holder 1 if it leads to profits

$$\hat{\Pi}_1 = [\alpha + (1-\alpha)(1-g(x_2))v]\,\bar{s}_1(s_2^*) > \alpha s_1^*.$$

This condition holds if

$$s_2^* < \sigma(G) \equiv \frac{(1-\alpha)v\left[\alpha(1-c) - G\left(1-\frac{c}{v}\right)\right] + G\frac{L_B}{1-g(x_2)}}{\alpha(G+(1-\alpha)v)},$$

where $G \equiv \alpha + (1-\alpha)(1-g(x_2))v \in [\alpha, 1]$. Thus, in instances in which $\sigma(G) < 1 - \frac{(1-2\alpha)cv+\alpha c}{(1-\alpha)v^2}$ an equilibrium with royalty stacking will fail to exist. This inequality implies that

$$G < G^*\left(\frac{L_B}{1-g(x_2)}\right) \equiv \frac{\alpha(1-\alpha)(1-v)cv(\alpha+(1-\alpha)v)}{-\frac{L_B}{1-g(x_2)}(1-\alpha)v^2 + (1-\alpha)v^2((1-\alpha)(v-c)+\alpha) - (1-2\alpha)\alpha cv - \alpha^2 c}.$$

This function is increasing in $\frac{L_B}{1-g(x_2)}$ and $G^*\left(\frac{L_B}{1-g(x_2)}\right) < G^*\left((\alpha+(1-\alpha)v) * \left(1 - \frac{(1-2\alpha)cv+\alpha c}{(1-\alpha)v^2}\right)\right) = \alpha + (1-\alpha)v$. Hence, there is always $g(x_2)$ sufficiently small so that the deviation will be optimal. □

C Equilibrium Litigation

A sustained assumption throughout the paper has been that it was always optimal for the innovators to avoid litigation. This assumption is consistent with high litigation costs of these upstream firms, L_U. In this section we analyze the implications of relaxing this assumption.

C.1 The Litigation Decision of Patent Holder 2

We concentrate in the situation where innovator 1 has a strong patent, $g(x_1) = 1$, whereas innovator 2's patent is weak, $g(x_2) < 1$. Figure 2 illustrates the structure of the game and defines the relevant payoffs. Whereas in the benchmark model we assumed that the downstream firm always accepted in equilibrium the royalty rate offered, here we need to

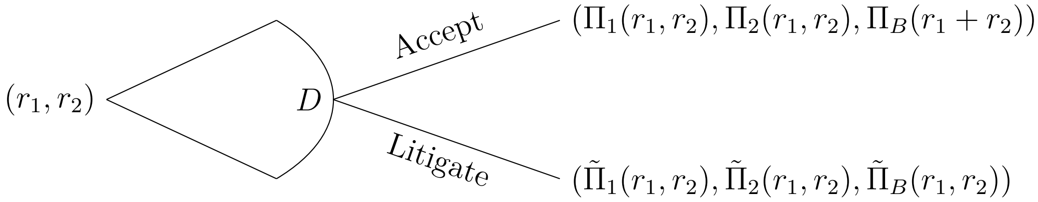

Figure 2: Timing of the game when litigation may occur in equilibrium.

define the payoffs when it prefers to go to court. These profits are defined as

$$\tilde{\Pi}_1(r_1, r_2) = g(x_2)\Pi_1(r_1, r_2) + (1 - g(x_2))\Pi_1(r_1, 0),$$

$$\tilde{\Pi}_2(r_1, r_2) = g(x_2)\Pi_2(r_1, r_2) - L_U,$$

$$\tilde{\Pi}_B(r_1, r_2) = g(x_2)\Pi_B(r_1 + r_2) + (1 - g(x_2))\Pi_B(r_1) - L_B,$$

where $\Pi_i(r_1, r_2)$ are the profits of innovator i when the downstream producer licenses both patents. The expression $\tilde{\Pi}_B(r_1, r_2)$ has been implicitly used before, since its difference with respect to $\Pi_B(r_1 + r_2)$ determines the results in Lemma 3.

From the previous expressions it is clear that, contingent on litigation, the optimal response of innovator 2 to a royalty r_1 coincides with the one that arises when litigation is not a threat. Denote this choice as $r_2^c(r_1)$ which, due to Assumption 1, is decreasing in r_1.

For a given r_1 we obtain the royalty rate that guarantees that it is not worthwhile for the downstream producer to litigate, $\bar{r}_2(r_1)$, as the inverse of $\bar{r}_1(r_2)$ in (1). Using Lemma 3, when L_B is sufficiently low, this function is increasing in r_1, L_B, and $g(x_2)$. Since $r_2^c(r_1)$ converges to 0 as r_1 grows, it is immediate that there is a threshold value $\hat{\rho}_1$ so that for $r_1 \geq \hat{\rho}_1$, $r_2^c(r_1) \leq \bar{r}_2(r_1)$. In that case, litigation would not be a relevant threat. When $r_1 < \hat{\rho}_1$, innovator 2 trades off the increase in revenues originated by $r_2^c(r_1)$ with the probability that the patent is invalidated, yielding a revenue of 0. The former will dominate if r_1 is sufficiently small, as the large decrease in the royalty rate necessary to fend off litigation is unlikely to be profitable.

In the rest of this section we rely on a specific demand structure to characterize the equilibrium. We consider the linear demand case, $D(p) = 1 - p$, where the downstream firm chooses a unique (monopoly) price. In that case $\tilde{D}(R) = \frac{1-R}{2}$ and, using the previous expressions, we can show that

$$r_2^c(r_1) = \frac{1 - r_1}{2}, \tag{11}$$

$$\bar{r}_2(r_1) = 1 - r_1 - \sqrt{(1 - r_1)^2 - \frac{4L_B}{1 - g(x_2)}}. \tag{12}$$

The next result characterizes the optimal royalty rate for innovator 2 in this case.

Proposition 7. *Suppose that demand is $\tilde{D}(R) = \frac{1-R}{2}$. When L_U is sufficiently low, there exists a unique threshold value $\tilde{\rho}_1(x_2, L_B, L_U) \in [0, \hat{\rho}_1)$ decreasing in L_B and L_U so that the optimal decision of innovator 2 becomes,*

$$r_2^*(r_1) = \begin{cases} r_2^c(r_1) & \text{if } r_1 < \tilde{\rho}_1, \\ \bar{r}_2(r_1) & \text{if } r_1 \geq \tilde{\rho}_1. \end{cases}$$

Proof. First notice that if $\frac{L_B}{1-g(x_2)} > \frac{3}{16}$ then $r_2^c(r_1) < \bar{r}_2(r_1)$ for all r_1. Otherwise, $r_2^c(r_1) < \bar{r}_2(r_1)$ if and only if $r_1 < \hat{\rho}_1 \equiv 1 - \sqrt{\frac{16L_B}{3(1-g(x_2))}}$.

Define $\tilde{\rho}_1$ as the value for which patent holder 2 will be indifferent between going to court or offering a royalty rate that the downstream producer will accept. That is,

$$\tilde{\Pi}_2\left(\tilde{\rho}_1, r_2^c(\tilde{\rho}_1)\right) = g(x_2)\Pi_2\left(\tilde{\rho}_1, r_2^c(\tilde{\rho}_1)\right) - L_U = \Pi_2\left(\tilde{\rho}_1, \bar{r}_2(\tilde{\rho}_1)\right),$$

where

$$\tilde{\Pi}_2(r_1, r_2^c) = g(x_2)\frac{(1 - r_1)^2}{8} - L_U,$$

$$\Pi_2(r_1, \bar{r}_2) = \frac{4L_B}{1 - g(x_2)} - (1 - r_1)\bar{r}_2(r_1).$$

We now show that this threshold is unique and litigation is preferred by patent holder 2 when $r_1 < \tilde{\rho}_1$. We can compute the effect of r_1 on both choices as

$$\frac{d\tilde{\Pi}_2}{dr_1}(r_1, r_2^c(r_1)) = -g(x_2)\frac{1 - r_1}{4} < 0,$$

$$\frac{d\Pi_2}{dr_1}(r_1, \bar{r}_2(r_1)) = -\frac{\bar{r}_2(r_1)^2}{2\sqrt{(1 - r_1)^2 - \frac{4L_B}{1-g(x_2)}}} < 0.$$

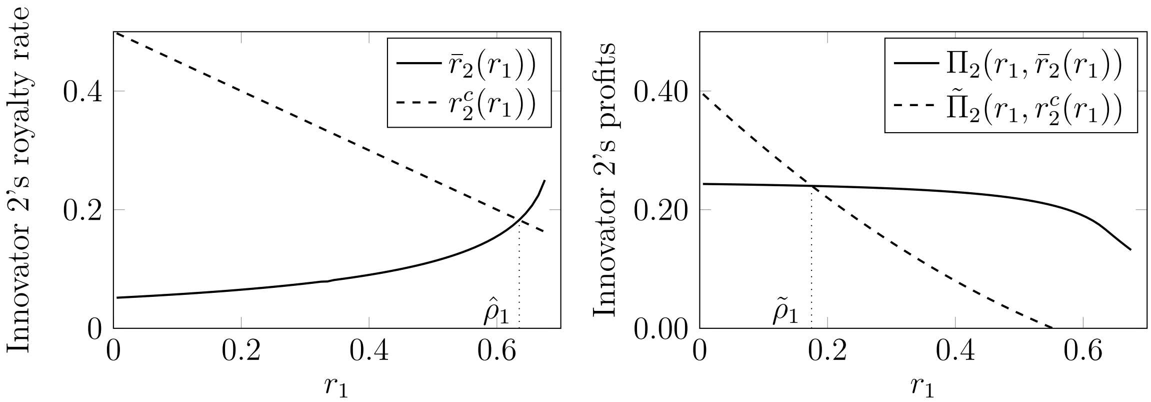

Figure 5: Royalties and profits under litigation and accommodation with parameter values: $L_B = 0.015$, $g(x_2) = 0.4$, $L_U = 0.01$.

Both derivatives are negative. However, notice that $\frac{d\tilde{\Pi}_2}{dr_1}(r_1, r_2^c)$ is increasing in r_1 whereas $\frac{d\Pi_2}{dr_1}(r_1, \bar{r}_2(r_1))$ is increasing in r_1. That is, $\tilde{\Pi}_2(r_1, r_2^c)$ is convex in r_1 and $\Pi_2(r_1, \bar{r}_2)$ is concave in r_1. This implies that there might be 0, 1 or 2 points in which these functions cross. We can rule out the case in which the functions cross twice, because $\tilde{\Pi}_2(\hat{\rho}_1, r_2^c(\hat{\rho}_1)) < \Pi_2(\hat{\rho}_1, r_2^c(\hat{\rho}_1))$ since in this case litigation does not imply a higher royalty rate. Hence, two possibilities remain: (i) the functions do not cross, which occurs if $\tilde{\Pi}_2(r_1, r_2^c(r_1)) < \Pi_2(r_1, r_2^c(r_1))$ for all values of r_1 or (ii) there is a single crossing point $\hat{\rho}_1 \in (0, \tilde{\rho}_1)$, which occurs if $\tilde{\Pi}_2(0, r_2^c(0)) > \Pi_2(0, r_2^c(0))$. The second case arises when L_U is sufficiently low as stated in the lemma.

The effect of L_U and L_B can be characterized directly from the derivatives

$$\frac{d\tilde{\Pi}_2}{dL_U}(r_1, r_2^c(r_1)) = -1,$$
$$\frac{d\Pi_2}{dL_B}(r_1, \bar{r}_2(r_1)) = \frac{1}{2} - \frac{1 - r_1}{4\sqrt{(1 - r_1)^2 - \frac{4L_B}{1 - g(x_2)}}}.$$

Obviously, the first expression is always negative. The second is positive if and only if r_1 is in the relevant range, $r_1 \leq \hat{\rho}_1$. □

This proposition shows that the optimal royalty rate of innovator 2 can be characterized by two regions. As explained before, when r_1 is low, it does not pay off for innovator 2 to decrease the royalty rate to avoid litigation. At the other extreme, when r_1 is sufficiently high, a small decrease in r_2 is required and avoiding litigation increases profits. Under the linear demand function, there is a unique threshold, identified as $\tilde{\rho}_1$, that determines the two regions. The discontinuity in profits of moving, for the same level of r_2, from a situation where litigation is averted to one in which it occurs explains why $\tilde{\rho}_1 < \hat{\rho}_1$.

Figure 5 illustrates this result. The left panel characterizes the threshold $\hat{\rho}_1$ which determines when the royalty choice of innovator 2 is restricted by litigation. The right panel show that the profits of innovator 2 are decreasing in r_1 with and without litigation but for different reasons. When there is no litigation in equilibrium $\Pi_2(r_1, \bar{r}_2(r_1))$ is decreasing in r_1 due to the standard Cournot arguments. When litigation occurs in equilibrium, however, this effect is moderated by the fact that the decrease in the royalty rate required to avoid litigation is now smaller. This counteracting effect also explains why the threshold for which litigation is preferred is unique.

This figure also allows us to illustrate how the threshold $\tilde{\rho}_1$ changes with the litigation costs.[4] Increases in L_U lead to a downward shift in $\tilde{\Pi}_2(r_1, \bar{r}_2^c(r_1))$ as litigation becomes less profitable for innovator 2. Similarly, in the case in which litigation does not occur, an increase in L_B allows innovator 2 to raise the royalty rate $\bar{r}_2(r_1)$, increasing $\Pi_2(r_1, \bar{r}_2(r_1))$. In both cases, $\hat{\rho}_1$ moves to the left and litigation is less likely to arise in equilibrium for a given value of r_1. The effect of an increase in x_2 is ambiguous since it raises profits in both cases.

C.2 The Optimal Choice of r_1

We now discuss how the optimal royalty rate of innovator 1 is affected by the possibility of litigation in equilibrium. To answer this question, we need to make an additional assumption. We depart from the structure of the baseline model and assume that innovator 1 moves first. As explained in section 5, this assumption has little impact in the case where one patent holder is strong and legal costs are high but it guarantees the existence

[4] It is immediate that $\hat{\rho}_1$ is decreasing in L_B and $g(x_2)$.

of an equilibrium in pure strategies. We continue to focus on the linear demand case discussed in the last proposition, which guarantees that litigation only emerges for low values of r_1.

From the previous analysis, we need to distinguish two cases. If $r_1 < \tilde{\rho}_1$ innovator 2 will choose $r_2^c(r_1)$ and litigation will arise in equilibrium. If $r_1 \geq \tilde{\rho}_1$, innovator 2 prefers to offer a lower royalty rate, $\bar{r}_2(r_1)$, and avoid being brought to court.

In the first case, innovator 1 maximizes

$$\max_{r_1 < \tilde{\rho}_1} r_1 \left[g(x_2)\tilde{D}(r_1 + r_2^c(r_1)) + (1 - g(x_2))\tilde{D}(r_1) \right],$$

which implies an equilibrium royalty rate $\tilde{r}_1^* = \min\left\{\tilde{\rho}_1, \frac{1}{2}\right\}$. Notice that when $\tilde{\rho}_1$ is sufficiently high, the royalty rate is a convex combination of the one that a monopolist and a Stackelberg leader would choose which, under a linear demand, coincide.

In the second case, a high r_1 is chosen and innovator 2 prefers to avoid litigation by setting a royalty rate $\bar{r}_2(r_1)$. Innovator 1 maximizes the profit function

$$\max_{r_1 \geq \tilde{\rho}_1} r_1 \frac{1 - r_1 - \bar{r}_2(r_1)}{2},$$

resulting in a candidate royalty rate $\bar{r}_1^* = \max\{\tilde{\rho}_1, r_1^{ic}\}$, where $r_1^{ic} \equiv \frac{3}{4} - \frac{1}{4}\sqrt{\frac{32L_B}{1-g(x_2)} + 1}$. Notice that $r_1^{ic} \leq \frac{1}{2}$ meaning that this royalty rate is lower than the unconstrained choice when litigation arises in equilibrium. That is, the unconstrained royalty rate of innovator 1 is lower in the case in which innovator 2 accommodates. This result is a version of the *Inverse-Cournot effect*: by reducing r_1 innovator 1 also fosters a reduction in r_2, mitigating the royalty-stacking distortion and increasing downstream sales and overall profits.

It can also be shown that if we, again, abstract from the constraints imposed by $\tilde{\rho}_1$, innovator 1's profits are always higher when $r_1 = r_1^{ic}$ and litigation does not occur in equilibrium compared to when $r_1 = \frac{1}{2}$ and innovator 2 is brought to court. The linear structure of the demand function implies that, for a given r_1, the option that maximizes profits is the one that leads to the lowest expected total royalty rate. This means that in some situations $r_1 = \tilde{\rho}_1$ will be optimal as a way to avoid litigation. The next result uses the previous insights to characterize the optimal royalty rate for innovator 1 as a function of the threshold $\tilde{\rho}_1$.

Proposition 8. *The optimal royalty rate of innovator* 1 *can be characterized as*

$$r_1^* = \begin{cases} r_1^{ic} & \text{if } \tilde{\rho}_1 \leq r_1^{ic}, \\ \tilde{\rho}_1 & \text{if } \tilde{\rho}_1 \in (r_1^{ic}, \rho^*], \\ \frac{1}{2} & \text{otherwise,} \end{cases}$$

where $\rho^* > \frac{1}{2}$.

Proof. To prove the result we only need to show that there are instances in which $\tilde{\rho}_1 > \frac{1}{2}$ so that $r_1 = 1/2$ would be feasible and it would induce litigation, but raising the royalty so that patent holder 2 would instead accommodate increases profits for patent holder 1.

First, notice that since $\tilde{\rho}_1 > \frac{1}{2}$ it has to be that $\tilde{\Pi}_2\left(1/2, r_2^c(1/2)\right) > \Pi_2\left(1/2, \bar{r}_2(1/2)\right)$. This condition implies that

$$g(x_2)\frac{1}{32} - L_U > \bar{r}_2(1/2)\frac{1/2 - \bar{r}_2(1/2)}{2}$$

or

$$\bar{r}_2(1/2) < \frac{1 - \sqrt{1 - g(x_2) + 32L_U}}{4}, \tag{13}$$

given that $\bar{r}_2(1/2) < r_2^c(1/2) = \frac{1}{4}$.

We now show that when $\tilde{\rho}_1 > \frac{1}{2}$, $\Pi_1\left(1/2, \bar{r}_2(1/2)\right) > \tilde{\Pi}_1\left(1/2, r_2^c(1/2)\right)$. This condition holds if the expected royalty rate is lower without litigation. That is, if

$$\frac{1}{2} + \bar{r}_2(1/2) < \frac{1}{2} + g(x_2)\frac{1}{4}.$$

This condition is satisfied given (13), since $\sqrt{1 - g(x_2) + 32L_U} > 1 - g(x_2)$.

By continuity, the previous conditions imply that if $\tilde{\rho}_1$ is sufficiently close to $\frac{1}{2}$ then $\Pi_1\left(\tilde{\rho}_1, \bar{r}_2(\tilde{\rho}_1)\right) > \tilde{\Pi}_1\left(1/2, r_2^c(1/2)\right)$. □

Figure 6 illustrates the optimal royalty rate for different values of $\tilde{\rho}_1$. This figure identifies two regions. For values of $\tilde{\rho}_1$ below a threshold ρ^* it is optimal for innovator 1 to induce a low royalty rate r_2 that will be accepted by the downstream producer. When

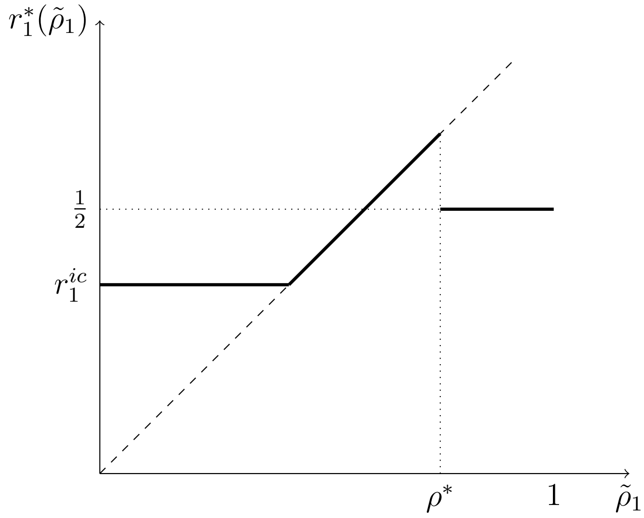

Figure 6: Optimal royalty rate of patent holder 1 as a function of $\tilde{\rho}_1$.

$\tilde{\rho}_1$ is higher than ρ^* inducing the litigation of patent 2 is optimal for innovator 1. The royalty rate that maximizes profits in that region is the unconstrained one.

In order to unpack the implications of the previous result, it is useful to illustrate the discussion by analyzing the effect of different values of L_U. The comparative statics exercise in this case is simple since the litigation cost of innovator 2 has no direct impact on the profits of innovator 1 except through the changes in $\tilde{\rho}_1$. From Proposition 7 we know that increases in L_U are associated with a decrease in $\tilde{\rho}_1$, as the higher the cost of innovator 2 to defend its patent in court the higher the royalty rate that innovator 1 can charge without triggering litigation. As it can be seen from the figure, when L_U is low, and therefore $\tilde{\rho}_1$ is high, innovator 1 is likely to find optimal to choose $r_1 = \frac{1}{2}$. The reason is that discouraging litigation (i.e. innovator 2 chooses a low r_2) would require a very high royalty rate. As a result the total burden $r_1 + \bar{r}_2(r_1)$ would become very high and the quantity sold low. As L_U increases, however, discouraging innovator 2 from litigating is easier and, eventually, when litigation costs are sufficiently high so that going to court is not a reasonable option, the Inverse-Cournot effect is the only relevant force. This effect pushes innovator 1 to choose a royalty rate lower than the one that would emerge when litigation was optimal. This case has been the focus of the main sections of the paper.

For intermediate values of L_U we observe a region in which litigation does not take place (to the left of ρ^*) but the royalty rate of innovator 1 is higher not only than r_1^{ic} but also than the one that would emerge under litigation. When $r_1^* = \tilde{\rho}_1$ the Inverse Cournot effect is relaxed, allowing innovator 2 to increase the royalty rate and making the option of avoiding litigation more profitable. Hence, for innovator 1, this higher royalty rate generates a trade-off. Choosing $\tilde{\rho}_1$, compared to $r_1 = \frac{1}{2}$, implies a higher individual royalty rate but a lower quantity due to the higher "expected" total royalty rate that emerges due to the lower probability that the patent of innovator 2 is invalidated.[5] For values of $\tilde{\rho}_1$ below ρ^*, this trade-off is resolved in favor of the high royalty rate even if that implies an increase in r_2.

The effect of L_B is similar in the sense that increases in this cost also shift the threshold value $\tilde{\rho}_1$ downwards. However, an increase in L_B also raises $\bar{r}_2(r_1)$, reducing the profits from discouraging innovator 2 to defend its patent in court. Both effects go in the same direction, suggesting that as L_B increases the region under which promoting litigation is optimal for patent holder 1 expands.

The effect of x_2 is in general difficult to ascertain, as it affects the figure in several dimensions. First, we can observe that, both under litigation and under accommodation, the profits of innovator 1 decrease as x_2 increase, since the problem of royalty stacking becomes more relevant. However, an analytical comparison of the magnitude of the effect in both cases as well as the effect of x_2 on $\tilde{\rho}_1$ is difficult to establish.

Finally, this example allows us to draw some implications for equilibrium royalty stacking. Trivially, when litigation emerges in equilibrium the expected royalty rate is lower than the one that arises when both patents are strong. The reason is that both innovators would choose the same royalty rate but the expected royalty rate decreases as the probability that the patent of innovator 2 is invalidated increases. At the other extreme, when $r_1^* = r_1^{ic}$ the Inverse Cournot effect implies that the resulting royalty rate

[5]For a given value of r_1 we can write the profits of patent holder 1 as $\Pi_1(r_1) = r_1\frac{1-R}{2}$, where $R = r_1 + g(x_2)r_2^c(r_1)$ and $R = r_1 + \bar{r}_2(r_1)$ when litigation occurs in equilibrium and when patent holder 2 avoids it, respectively.

is lower than the one that would emerge under ironclad patents.

Interestingly, in the intermediate region, when $r_1^* = \tilde{\rho}_1$ and litigation is credible, the implications for royalty stacking are ambiguous. Innovator 1 can increase revenues by raising the royalty rate above $\frac{1}{2}$ even as this fosters a limited increase in r_2. Under some parameter configurations this may lead to a higher total royalty rate.